A Rational, Decentralized Ponzi Scheme

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Abstract

We present a model of an industry with a dynamic, monopoly financial firm and an OLG population of depositors. The firm is a special case of the banking firm in Carpio (2011): it is a bank without a lending business, i.e. a Ponzi scheme. The firm offers an interest rate on deposits; depositors are forward-looking and take the interest rate and bank risk as given, and choose how much to save using simple mean-variance preferences. Interest rates, financial risk, and quantities of credit are endogenous. We solve the model numerically through value function iteration. Numerical experiments show that an equilibrium with nonzero savings can exist if depositor risk aversion and population growth uncertainty is not too high; bank failure may occur optimally in finite time.

1 Introduction

The financial crisis of 2007-2009 demonstrated that financial intermediaries play a critical, if not yet well-understood, role in the economy. When economists want to understand a phenomenon, they turn to their models; many of our workhorse economic models, however, are not well suited for analyzing banks and other intermediaries. For example, the standard general equilibrium model has no role for banks; the standard representative agent macro model has no role for quantities of debt.

In Carpio (2011) we presented a dynamic model of a banking firm based on inventory management of stochastic cash flows. That paper developed a theoretical model of a bank that takes deposits, makes loans, and engages in maturity mismatch. Cash inflows were generated by new deposits and repayment of loans; cash outflows were generated by withdrawal of deposits, new loans being made, and dividend payouts. A limitation of that paper was that interest rates and the supply of deposits (i.e. the quantity of cash inflows from new deposits) were exogenous. It would be valuable if we could endogenize interest rates and quantities of credit by specifying the decision problem of the bank's customers; doing this for both borrowers and lenders would allow us to analyze both prices (e.g. interest rates and spreads) and quantities (the amount of savings, which influences consumption; the amount of lending, which influences investment) in equilibrium.

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This is obviously an ambitious task; this paper takes a small step towards this goal by developing a very simple model of an industry with an inventory-theoretic financial firm. A Ponzi scheme can be thought of as a bank without a lending business; the only source of cash inflows are new depositors, while cash outflows are generated by withdrawals of deposits and dividend payouts. While real-life Ponzi schemes are usually fraudulent, it is possible to imagine a "rational" Ponzi scheme, in which depositors with rational expectations willingly invest money. We model a single monopoly firm that offers a price (i.e. interest rate) to an overlapping-generations population of customer/depositors. Depositors live two periods. When young, they are endowed with an income and make a portfolio choice decision of how much to hold in riskless cash, and how much to save in risky bank deposits.

The aggregate amount of savings in a period translates into a cash inflow for the bank. When old, depositors will withdraw their deposits at the promised interest rate, which translates into a cash outflow for the bank. Population growth is stochastic; from the bank's point of view, this translates into uncertainty in cash inflows. The bank chooses its dividend payout and interest rate each period; it may go bankrupt, in which case new depositors will lose everything, resulting in a zero return on their savings. From the depositors' point of view, deposits are a risky investment; they take this risk into account when choosing how much to save. Equilibrium is defined in the IO sense, when the firm takes the expected demand schedule as given and jointly chooses price and quantity supplied, while consumers take price as given and choose quantity consumed.

While obviously unrealistic and simplified, this model attempts a novel formalization of a number of aspects of the financial industry. First, we model financial assets as something produced by a firm, which is capable of choosing an offering price and producing more or less (subject to demand), while also behaving as an asset with a well-defined risk and return. In this way we attempt to connect the well-developed theory of savings and portfolio choice on the consumer side, with the theory of production by a profit-maximizing firm.

Second, we endogenize the risk of bank failure, while also having depositors be forward-looking. Third, it is a truism when discussing debt to say that "every asset is a liability to someone else", but it is not often explicitly modeled. In our model, this arises from the fact that cash flows that are owed to a depositor (i.e. an asset, from the depositor's point of view) must depart in the same amount from the bank (i.e. a liability).

Finally, our model is similar to OLG models of social security in which the young of one generation transfer some wealth to the old of the previous generation, but the actual mechanism by which this takes place is usually assumed to be a welfare-maximizing government, or sometimes a passive financial intermediary. We explicitly detail this mechanism, and decentralize it in two ways: first, in contrast to taxes or forced savings, depositors choose if and how much to invest. Second, the intermediary's objective function is explicitly profit maximization.

2 Related literature

The notion of banking as the inventory management of cash goes back to Edgeworth (1888). Carpio (2011) discusses the basic model used in this paper and its relation to the literatures on inventory management and the "optimal dividends" problem of insurance introduced by de Finetti (1957).

Overlapping-generations models are commonly used to model public pensions and Social Security-like schemes; if there is population or income growth, then a positive interest rate can be sustained in Walrasian competitive equilibrium (Geanakoplos 1987). As in standard models of general equilibrium, there is no role for financial intermediaries. Qi (1994) extends the Diamond and Dybvig (1983) model of banks as liquidity insurance providers to an OLG model; as in that paper, the bank is not modeled as a self-interested agent, but rather as maximizing the welfare of its depositors. In our model, the bank is a profit-maximizing firm, and we show how intermediation (and possible failure) arises from optimal behavior.

Artzrouni (2009) models a Ponzi scheme with a first-order differential equation, and gives conditions that the deposit and withdrawal rates must satisfy in order to avoid collapse; in contrast to this paper, the decision problems of the bank and its investors are not modeled.

For depositor preferences, we use a simple version of the mean-variance formulation originated by Markowitz (1952). This is for convenience; this results in a simple portfolio choice decision problem that we believe is still meaningful. More complex preferences can be captured by a different mean-variance indifference curve, or by directly solving a fully specified consumption-savings problem.

3 The Model

3.1 Environment

The population of households is a 2-period OLG with stochastic population growth. Let N_t denote the number of births at time t. Then population growth between t and $t+1$ will be $\frac{N_{t+1}}{N_t} = g_t$, which is a random variable. Assume that g_t can take on two values g_{low}, g_{high} with probability $1 - p_{high}, p_{high}$ respectively.

3.2 Bank's problem

The bank begins each period with the following state variables: starting cash reserve M_t, maturing liabilities D_t, and current population of depositors (i.e. households in old age) N_t. The probability distribution of g_t and the depositor and bank's problems are common knowledge. Assume the bank moves first by choosing the dividend $d_t \in [0, M_t]$ and the offered interest rate on deposits $r_t \geq 0$. The depositors then choose how much to invest in bank deposits, given the bank's state and action, denoted $f(r, M, D, N)$. The depositor's problem will be specified below; we assume $f(\cdot)$ is known to the bank. The bank behaves strategically with respect to the depositor (i.e. it takes the depositor's reaction into account when choosing d_t, r_t). In each period, the sequence of events is as follows:

1. Bank chooses $d_t \in [0, M_t], r_t \in [0, \infty)$.

2. D_t, d_t is paid out; at this point in time, the previous period's investors are certain to get paid, and the bank will receive this period's utility $u(d_t) = d_t$

3. population growth $g_t = N_{t+1}/N_t$ is realized, but not observed by bank or depositors

4. newly born depositors choose savings, causing cash inflow $f(r_t, M_t, D_t, N_t)N_{t+1}$

5. g_t is revealed to everyone. Calculate $M_{t+1} = M_t + f(r_t, M_t, D_t, N_t)N_{t+1} - D_t - d_t$

6. if $M_{t+1} \leq 0$, the bank fails, ceases operating, and receives a zero payout in all future periods. The bank also experiences a utility penalty ZM_{t+1} that is linear in the amount of the shortfall. This period's depositors will get a zero return. Otherwise, begin next period with $M_{t+1}, D_{t+1} = (1 + r_t)f(r_t, M_t, D_t, N_t)N_{t+1}$

Note that cash is not conserved if the bank fails; total outflows $D_t + d_t$ will exceed M_t. It is possible to eliminate this problem by going to continuous time (which would eliminate discontinuous jumps past zero) or by modeling partial recovery of assets in bankruptcy. Alternatively, we could make the payout of d_t conditional on survival, in which case cash would be destroyed in bankruptcy. In the interest of simplicity, we will assume that there is an outside insurance fund that ensures previous depositors get paid. The Bellman equation for this problem is:

$$V(M_t, D_t, N_t) = \max_{r_t, d_t} \{d_t + \beta E [V(M_{t+1}, D_{t+1}, N_{t+1})]\}$$

subject to boundary condition $V(M \leq 0, \cdot, \cdot) = -ZM$ and constraints

$$M_{t+1} = M_t + f(r_t, M_t, D_t, N_t)N_{t+1} - D_t - d_t$$

$$D_{t+1} = (1 + r_t)f(r_t, M_t, D_t, N_t)N_{t+1}$$

$$N_{t+1} = g_t N_t$$

Since $u(d_t) = d_t$ and $V(M \leq 0, \cdot, \cdot) = -ZM$ are linear functions, we can eliminate N_t from the state variables by dividing M_t, D_t, and d_t by N_t: let $\overline{d_t} = \frac{d_t}{N_t}, \overline{D_t} = \frac{D_t}{N_t}, \overline{M_t} = \frac{M_t}{N_t}$. These can be thought of as quantities per "current customer". Substituting these variables into the problem, we can define an equivalent problem with Bellman equation

$$\overline{V}(\overline{M_t}, \overline{D_t}) = \max_{r_t, \overline{d_t}} \left\{\overline{d_t} + \beta E \left[g_t \overline{V}(\overline{M_{t+1}}, \overline{D_{t+1}})\right]\right\}$$

subject to boundary condition $\overline{V}(\overline{M} \leq 0, \cdot) = -Z\overline{M}$ and constraints

$$\overline{M_{t+1}} = \frac{1}{g_t}(\overline{M_t} + f(r_t, \overline{M_t}, \overline{D_t})g_t - \overline{D_t} - \overline{d_t}) \tag{1}$$

$$\overline{D_{t+1}} = (1 + r_t)f(r_t, \overline{M_t}, \overline{D_t}) \tag{2}$$

The interpretation of these constraints are as follows:

1. cash reserve per customer at $t + 1$ = net cash after inflows/outflows per customer at t, shrunk by g_t

2. liabilities per customer at $t+1$ = amount deposited in t plus interest

The value function of this problem is the "per-customer" fraction of the previous value function: $\overline{V}(\overline{M_t}, \overline{D_t}) = \frac{V(M_t, D_t, N_t)}{N_t}$. For some parameter values (e.g. if $\beta g_{low} > 1$) it will be optimal in each period to retain all cash and pay no dividend; an optimal policy may not exist in these cases.

3.3 Depositor's problem

We make the following assumptions:

- Depositors live 2 periods. In the first period, they receive an income of 1, all of which must be invested, and face a portfolio allocation problem that determines how much wealth they will have in the second period.
- Depositors can save via two assets: a riskless storage asset with a return of 1 (a cash-like asset), and bank deposits, which are risky. The bank offers interest rate r, but can go bankrupt, in which case nothing is returned. If the probability that the bank does not go bankrupt is ρ, then returns on bank deposits follow a Bernoulli distribution scaled by $(1+r)$, with mean $(1+r)\rho$ and variance $(1+r)^2\rho(1-\rho)$.
- Preferences over wealth are of mean-variance type. We assume a convenient form: $u(z) = E(z) - m \cdot SD(z)$, where m captures the degree of "risk aversion". The indifference curves for this representation are parallel lines in SD-mean space.
- Depositors are borrowing and liquidity constrained: they cannot go short cash or bank deposits.
- If depositors face a "tie" between cash and bank deposits, they will choose bank deposits.
- Depositors behave strategically with respect to their actions; that is, they know how their choice affects the bank and take that into account.

The efficient frontier for a portfolio of one riskless and one risky asset and with borrowing/liquidity constraints is a line segment with endpoints at the (mean, SD) point of each individual asset. Let f denote the fraction of the total income=1 that is invested in the risky asset. If the slope of the frontier is greater than m, the depositor will put everything into the risky asset ($f = 1$); otherwise, the depositor will hold everything in the riskless asset ($f = 0$). Suppose the depositor knows the bank's state variables $\overline{M}, \overline{D}$ and its choices $\overline{d}, r$ for this period. The bank will survive if $\overline{M_{t+1}} = \frac{1}{g_t}(\overline{M_t} + f g_t - \overline{D_t} - \overline{d_t}) > 0$, or if $f g_t > \overline{d_t} - \overline{M_t} + \overline{D_t}$. Denote $k = \overline{d_t} - \overline{M_t} + \overline{D_t}$, the per-customer net cash outflow before new deposits. The depositor knows that his choice of f determines the cash inflow to the bank and therefore its probability of survival, denoted $\rho(f)$. By assumption, if the depositor can choose f to cause a "self-fulfilling prophecy" resulting in a favorable return, he will do so. Since f will only take on values 0 or 1, we need only examine $\rho(0)$ and $\rho(1)$. Recall that $g_t \in \{g_{low}, g_{high}\}$ with probabilities $1 - p_{high}, p_{high}$ respectively.

- If $k < 0$, the bank will survive with certainty even if $f = 0$. Therefore, optimal $f = 1$.
- If $k > g_{high}$, the bank will fail with certainty even if $f = 1$. Therefore, optimal $f = 0$.

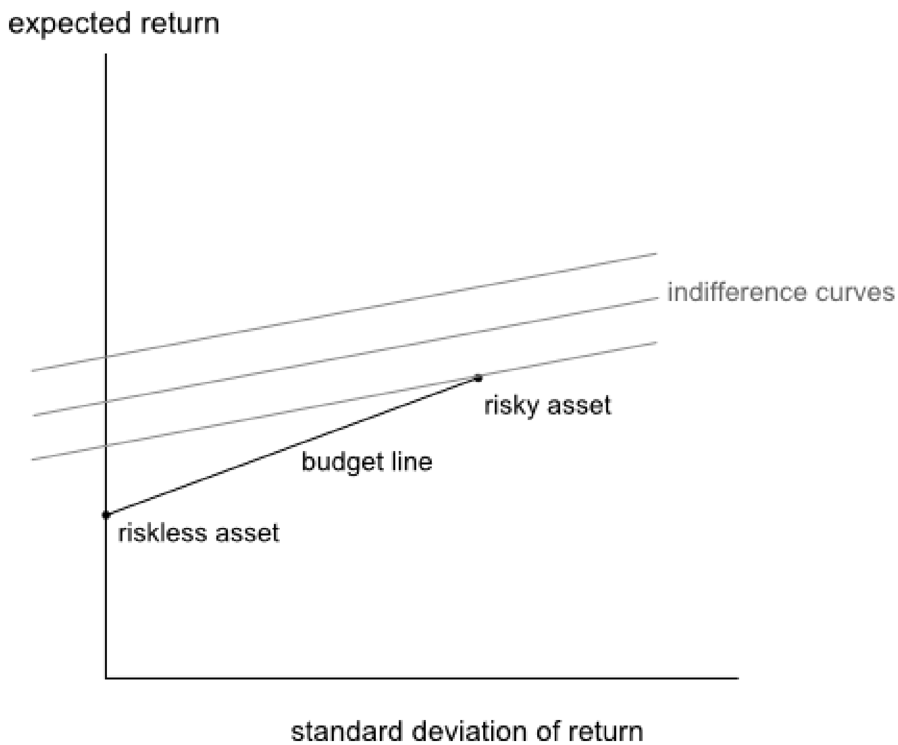


Figure 1: Depositor's portfolio choice problem

- If $k \leq g_{low}$, the bank will fail with certainty if $f = 0$ and survive with certainty if $f = 1$. Therefore, optimal $f = 1$.
- If $g_{low} < k \leq g_{high}$, the bank will fail with certainty if $f = 0$, but will survive with a good shock if $f = 1$: $\rho(0) = 0, \rho(1) = p_{high}$. Therefore, the depositor will choose $f = 1$ if the slope of the efficient frontier using a Bernoulli probability of p_{high} is greater than the risk-aversion parameter m:

$$m \leq \frac{(1+r)p_{high} - 1}{(1+r)\sqrt{p_{high}(1-p_{high})}}$$

The depositor's portfolio choice problem is graphically shown in mean-SD space in Figure 1.

4 Numerical solution

Now that we have $f(r_t, \overline{M_t}, \overline{D_t})$, we have all the parts needed for a solution to the bank's problem. We solve the problem numerically using value function iteration.[1] For the benchmark problem, we assume the following parameter values:

- Stochastic population growth: $g_t \in \{0.8, 1.2\}$ with probability 0.1, 0.9 respectively.

[1]The computational procedure was implemented using C++ and Python and run on a 2.8 GHz quad-core Intel CPU. The benchmark case took about 13 minutes to converge.

- Depositor risk aversion: $m = 0.1$
- Bank's discount rate: $\beta = 0.8$
- Utility penalty for bankruptcy is zero: $Z = 0$
- M_t, D_t space is approximated by a regular 160x200 grid, taking values between 0 and 4 for both M_t and D_t.

The computed value function $V(M_t, D_t)$ is shown in Figure 2. The optimal policies $d_t^*(M_t, D_t)$ and $r_t^*(M_t, D_t)$ are shown in Figure 3. Finally, we show M_{t+1}^*, the optimal choice for next period's beginning cash reserve, conditional on g_{high} occurring in Figure 4; if g_{low} occurred then M_{t+1} is simply shifted downward. In these graphs, M_t and D_t are on the X and Y axes.

Consider Figure 2, which shows the value function. We color each grid point based on the probability of bankruptcy and the optimal policy function. Starting from the origin along the direction of increasing D_t, the properties of these points are:

- Yellow: the bank survives with certainty, and $d_t^* = M_t$. The bank extracts all cash reserves, but survival is still assured due to cash inflows of $f = 1$ from the depositors. The bank does not offer an interest rate above that of cash, so $r_t^* = 0$.
- Green: the bank survives with certainty, and $d_t^* < M_t$. The bank does not offer an interest rate above that of cash, so $r_t^* = 0$.
- Blue: the bank survives if g_{high} occurs and cash inflows are high; it goes bankrupt otherwise. Since bank deposits are now a risky asset, the bank must offer a higher return than cash; $r_t^* > 0$. For the benchmark parameter values, $r_t^* = 0.162$.
- Red: the bank will go bankrupt with certainty, and $d_t^* = M_t$ (bank extracts all cash). r_t^* is irrelevant, since the depositor will not invest at any rate.

Note that in the red, green, and yellow regions, the slope of V along the M_t axis is 1; this is because an increase in M_t directly translates into an increase in cash extracted. In the red region, the slope of V along the D_t axis is zero, since additional liabilities are irrelevant when bankruptcy is certain. The slope is 1 in the green region, since additional M_t will be extracted; and it is less than 1, depending on β, in the yellow region, where additional M_t will go towards increasing M_{t+1} in the next period.

We can also examine the dynamic properties of this system. The initial values of (M_t, D_t) and the transition functions induced by the optimal policies define a Markov chain; we approximate this chain by taking each grid point as a state, which transitions to the grid point closest to (M_{t+1}, D_{t+1}) conditional on g_{low} or g_{high} occurring. If $M_{t+1} < 0$, the chain transitions into an absorbing bankruptcy state. Bankruptcy will occur in finite time almost surely, but we are interested in how long the bank survives before going bankrupt; in particular, we would like to know if the states in which the bank offers $r_t > 0$ can reach themselves with positive probaility. If this is not possible, then then in a sense, the world (as specified by our behavioral and stochastic parameters) cannot support this model of banking, even given arbitrarily unlikely sequences of good shocks.

From the graph induced by our approximated Markov chain, we can compute the set of states that can reach themselves with positive probability; we can also compute the expected hitting

Figure 2: Value function for $g_t \in \{0.8, 1.2\}, p_{High} = 0.9, m = 0.1, \beta = 0.8, Z = 0$

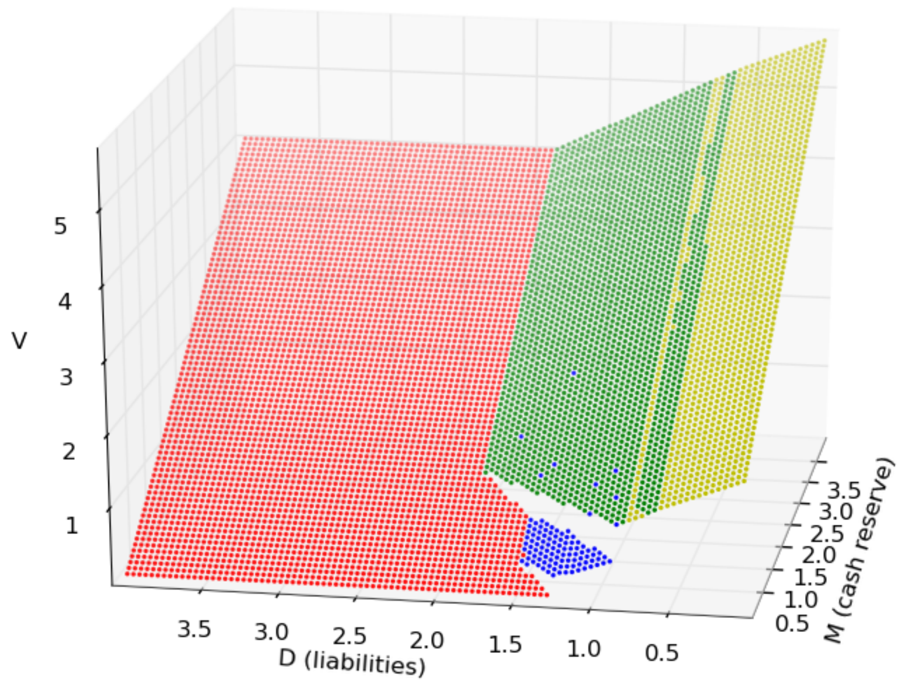


Table 1: Transition matrix and expected hitting time of bankruptcy state

(M_t, D_t)	•0.050, 1.003	•0.323, 1.003	•0.348, 1.003	•0.050, 1.162	•0.174, 1.162	bankruptcy	E(hitting time)
•0.050, 1.003	0	0	0	0	0.9	0.1	10
•0.323, 1.003	0.1	0	0.9	0	0	0	20
•0.348, 1.003	0.1	0.9	0	0	0	0	20
•0.050, 1.162	0	0	0	0.9	0	0.1	10
•0.174, 1.162	0	0	0	0.9	0	0.1	10

time of the absorbing bankruptcy state. Table 1 shows the transition matrix of grid points that remain after repeatedly removing nodes with zero indegree (i.e. cannot be reached from any previous state). There are three blue nodes, which will transition into bankruptcy if g_{low} occurs with probability 0.1; otherwise, the system will transition into another blue node. We can calculate the expected time to bankruptcy for each state with the transition matrix; the blue nodes are one bad shock away from bankruptcy, and therefore survive with an expected time of $1/p_{low} = 1/0.1 = 10$. Green states remain green until a bad shock occurs, then transition to blue; with a total expected hitting time of 20.

4.1 Comparative statics

We numerically examine the effects of changes in parameter values. Solutions for different values of $\beta, p_{high}, m, g_{low}, g_{high}$, and Z were computed, starting with the values given above as the baseline. We focus on two outcomes of interest:

Figure 3: Optimal policies d_t^* and r_t^* for $g_t \in \{0.8, 1.2\}, p_{High} = 0.9, m = 0.1, \beta = 0.8, Z = 0$

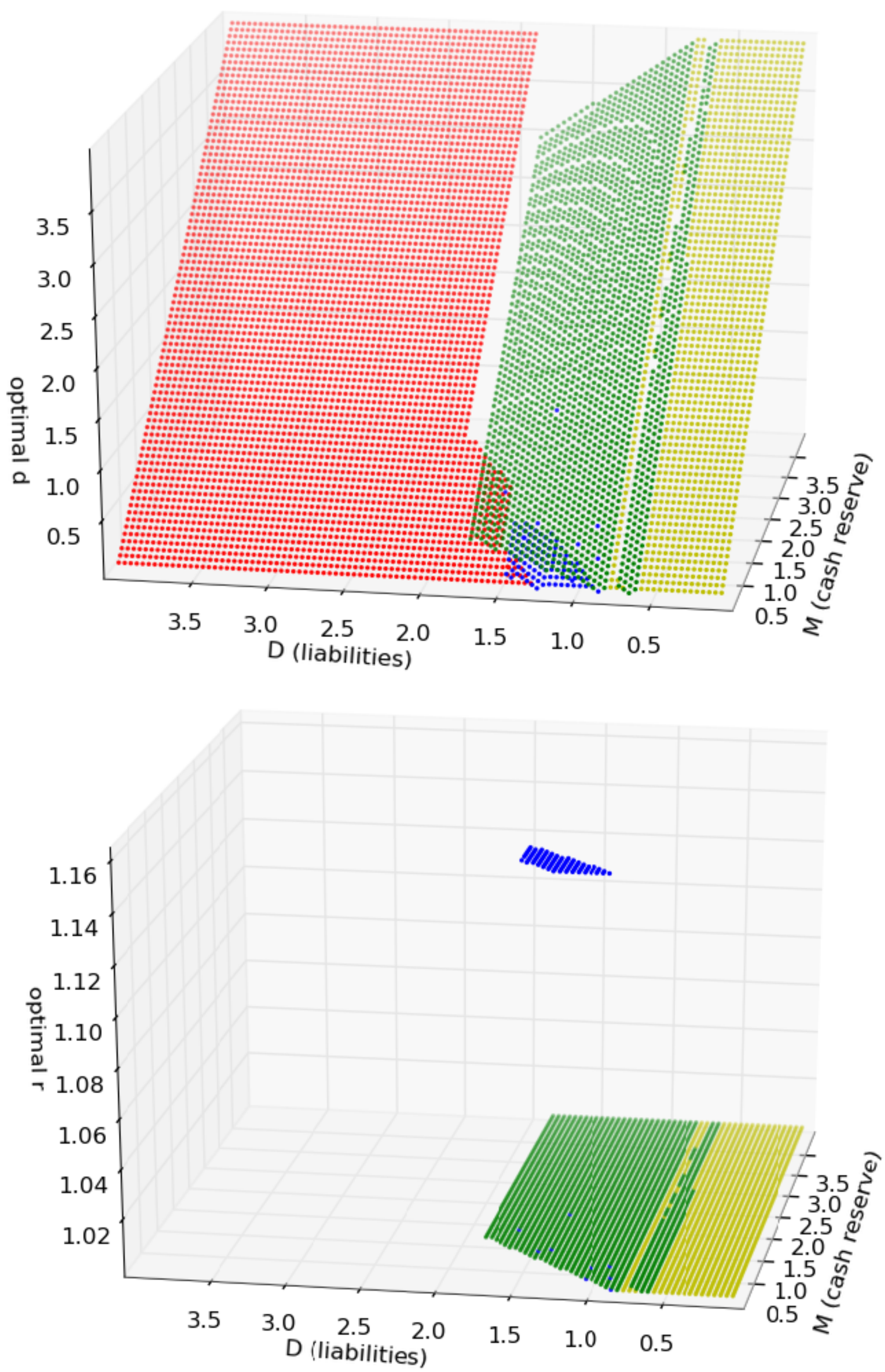
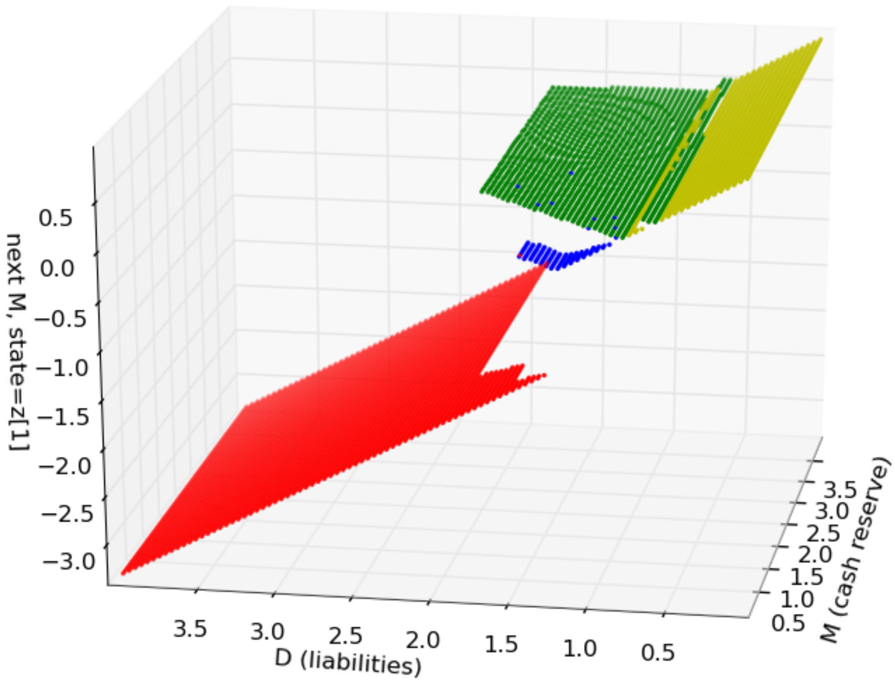


Figure 4: M_{t+1} conditional on g_{high} for $g_t \in \{0.8, 1.2\}, p_{High} = 0.9, m = 0.1, \beta = 0.8, Z = 0$



1. *Interest rate offered.* The only parameters that are found to affect r_t^* are p_{high} and m; this is because the only parameters that enter into the depositor's decision are the interest rate, probability of bankruptcy, and risk aversion, and because the bank is a monopolist. A higher population growth rate, which results in a higher equilibrium interest rate in other OLG models, is "captured" by the bank as increased profits. Figures 5 and 6 show r_t^* for different values of p_{high} and m. As the probability of a high population shock increases, bank deposits become less risky, and the bank can offer a lower interest rate while still attracting deposits. Similarly, a higher risk aversion parameter means the bank must offer a higher interest rate.

2. *Expected time to bankruptcy.* For a given set of parameters, we compute the set of states that can reach each other with positive probability, the transition probabilities between them, and their expected time to hitting the absorbing bankruptcy state. Table 2 reports the highest expected hitting time for a range of parameters, starting from the baseline. Some parameter values result in a chain that hits bankruptcy in bounded time for any initial state; these are reported as "None". Other chains can have states that never hit bankruptcy; these are reported as ∞. A higher β, lower depositor risk aversion, and higher population growth increase the expected time to bankruptcy. Varying the bankruptcy penalty did not appear to have an effect for the range of parameter values tested here.

Figure 5: Interest rate offered vs. probability of high population shock

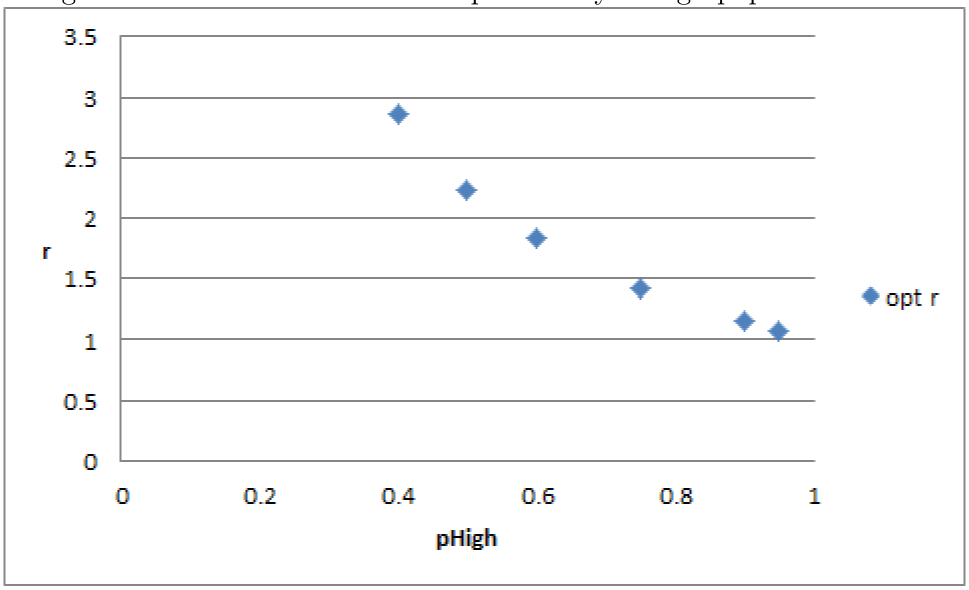


Figure 6: Interest rate offered vs. depositor risk aversion

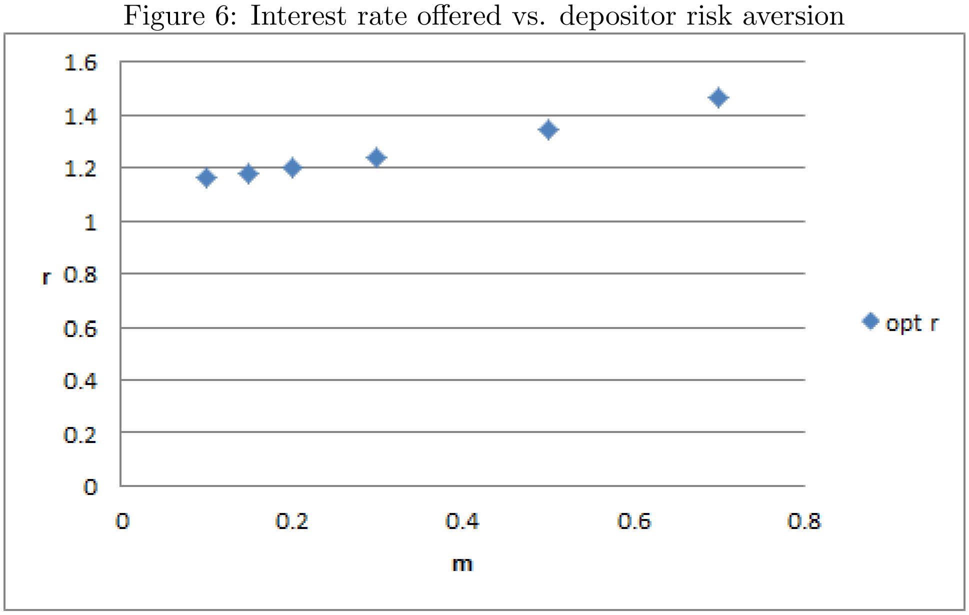


Table 2: Maximum expected hitting time of bankruptcy state

β	p_{high}	z_{low}	z_{high}	m	Z	max E(hitting time)
0.8	0.9	0.8	1.2	0.1	0	20
0.7	0.9	0.8	1.2	0.1	0	20
0.6	0.9	0.8	1.2	0.1	0	10
0.5	0.9	0.8	1.2	0.1	0	10
0.8	0.95	0.8	1.2	0.1	0	160.22787
0.8	0.9	0.8	1.2	0.1	0	20
0.8	0.75	0.8	1.2	0.1	0	5.75
0.8	0.6	0.8	1.2	0.1	0	None
0.8	0.9	0.5	1.2	0.1	0	10
0.8	0.9	0.6	1.2	0.1	0	10
0.8	0.9	0.7	1.2	0.1	0	20
0.8	0.9	0.8	1.2	0.1	0	20
0.8	0.9	0.9	1.2	0.1	0	120
0.8	0.9	1	1.2	0.1	0	∞
0.8	0.9	0.8	1	0.1	0	None
0.8	0.9	0.8	1.1	0.1	0	11.9
0.8	0.9	0.8	1.2	0.1	0	20
0.8	0.9	0.8	1.3	0.1	0	310.782204
0.8	0.9	0.8	1.2	0.1	0	20
0.8	0.9	0.8	1.2	0.15	0	20
0.8	0.9	0.8	1.2	0.2	0	20
0.8	0.9	0.8	1.2	0.3	0	20
0.8	0.9	0.8	1.2	0.5	0	11.9
0.8	0.9	0.8	1.2	0.7	0	11.9
0.8	0.9	0.8	1.2	0.1	0	20
0.8	0.9	0.8	1.2	0.1	-0.1	20
0.8	0.9	0.8	1.2	0.1	-0.2	20
0.8	0.9	0.8	1.2	0.1	-0.5	20
0.8	0.9	0.8	1.2	0.1	-0.7	20
0.8	0.9	0.8	1.2	0.1	-1	20

5 Conclusion and future research

We have presented a model of a banking firm as an inventory manager of cash flows, and shown that this model is general enough to apply to many other types of financial intermediaries. A Ponzi scheme is a simple instance of our model in which deposits from new investors are the only cash inflow; we numerically solved a partial equilibrium model of an industry with a single monopoly "bank". We showed that for a benchmark set of parameter values, the optimal value functions and optimal policies partition the state space into "bankrupt", "safe", and "risky" regions; the latter region in characterized by the bank paying nonzero interest. Markov chain analysis allows us to calculate the expected time the bank survives before entering bankruptcy. Numerical comparative statics shows that decreasing the probability of the good population shock, as well as increasing the depositor's risk aversion, increases the equilibrium interest rate up to a certain point, beyond which no interest above cash is offered at all.

This model is fairly simple and can be extended in interesting ways. First of all, we can add the lending side of the business, which would allow us to examine an endogenously chosen maturity structure. We would also be able to examine effects on one side of the business (e.g. lending) caused by changes in parameters to the other side (e.g. changes in rates at which the bank borrows).

We can also introduce competition into the model, and examine how competitive forces affect equilibrium interest rates, bank profits, and the risk of bankruptcy. Finally, we can examine other financial intermediaries (e.g. insurance, pension funds) using this approach.

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