Advanced Microeconomic Analysis, Lecture 6

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Administrative Stuff

- Homework #2 is due at the end of class. I will post the solutions on the website later today.
- The midterm will next week, on April 17.
- Midterm will be open-book.
- Chapters 1, 2.1, 2.4, 3, and 4.1-4.2 (up to monopoly) will be covered.
- Sample midterms from previous semesters are on the course website.

Review of Last Week

- A firm takes inputs and converts it into an output.
- The *production function* specifies how much output can be produced with a given combination of inputs (e.g. capital, labor).
- The production function is analogous to the utility function in consumer theory.
- We assume it is differentiable, strictly increasing, strictly quasiconcave.
- An *isoquant* is the set of inputs that produce a given level of output (like indifference curves).
- A *isocost* line is the set of inputs that cost the same amount (like a budget line).

Review of Last Week

- The *elasticity of substitution* measures the response of the MRTS (i.e. slope of the isoquant) as the ratio of input changes.
- *Returns to scale* determine how output responds when inputs are multiplied by a constant t. Returns may be:
  - Decreasing: output is increased by less than a multiple of t.
  - Constant: output is increased by a multiple of t.
  - Decreasing: output is increased by more than a multiplee of t.
- If the production function is homogeneous of degree d, then d determines the returns to scale ($d < 1 \rightarrow$ decreasing; $d = 1 \rightarrow$ constant; $d > 1 \rightarrow$ increasing)

Review of Last Week

- Given input prices w_1, w_2 and output level y, the *cost function* is the optimal value of the problem

$$c(\boldsymbol{w}, y) = \min_{\boldsymbol{x} \in \mathbb{R}^n_+} \boldsymbol{w} \cdot \boldsymbol{x} \qquad \text{s.t.} \qquad f(\boldsymbol{x}) \geq y$$

- The solution to this problem, $\boldsymbol{x}^*(w_1, w_2, y)$ is called the *conditional input demand* function.
- The cost function is analogous to the expenditure function in consumer theory, and the conditional input demand is analogous to Hicksian demand.
- All the properties and relationships between the two are similar.

Short-Run Profit Maximization

- As before, the long-run profit function is when all inputs can be changed. The short-run profit function is when some inputs must be fixed.
- Theorem 3.9: Suppose that f is continuous, strictly increasing, and strictly concave.
  - For $k < n$, let $\overline{\boldsymbol{x}} \in \mathbb{R}^k_+$ be a subvector of fixed inputs
  - Consider $f(\boldsymbol{x}, \overline{\boldsymbol{x}})$ as a function of the subvector of variable inputs $\boldsymbol{x} \in \mathbb{R}^{n-k}_+$.
  - Let $\boldsymbol{w}, \overline{\boldsymbol{w}}$ be the vector of prices for the variable and fixed inputs, respectively.
  - The *short-run*, or *restricted*, profit function is:

$$\pi(p, \boldsymbol{w}, \overline{\boldsymbol{w}}, \overline{\boldsymbol{x}}) = \max_{y, \boldsymbol{x}} py - \boldsymbol{w} \cdot \boldsymbol{x} - \overline{\boldsymbol{w}} \cdot \overline{\boldsymbol{x}} \qquad \text{s.t. } f(\boldsymbol{x}, \overline{\boldsymbol{x}}) \geq y$$

- The solutions $y(p, \boldsymbol{w}, \overline{\boldsymbol{w}}, \overline{\boldsymbol{x}}), \boldsymbol{x}(p, \boldsymbol{w}, \overline{\boldsymbol{w}}, \overline{\boldsymbol{x}})$ are called the short-run output supply and variable input demand functions.

Short-Run (or Restricted) Profit Function

- For all $p > 0$ and $\boldsymbol{w} >> 0, \pi(p, \boldsymbol{w}, \overline{\boldsymbol{w}}, \overline{\boldsymbol{x}})$, where well-defined, is:
  - continuous in p and $\boldsymbol{w}$,
  - increasing in p
  - decreasing in $\boldsymbol{w}$
  - convex in $(p, \boldsymbol{w})$
  - If $\pi(p, \boldsymbol{w}, \overline{\boldsymbol{w}}, \overline{\boldsymbol{x}})$ is twice continuously differentiable, the short-run output supply and variable input demand functions have the same properties as in Theorem 3.8 (homogeneity of degree zero, own-price effects, and positive semidefinite substitution matrix)

Optimal Shutdown

- Recall that $sc(p, \mathbf{w}, \overline{\mathbf{w}}, \overline{\mathbf{x}})$ is the short-run cost function. Consider the short-run profit function

$$\pi(p, \mathbf{w}, \overline{\mathbf{w}}, \overline{\mathbf{x}}) = max_y py - sc(p, \mathbf{w}, \overline{\mathbf{w}}, \overline{\mathbf{x}})$$

- The first-order condition tells us that for optimal $y^* > 0$,

$$p = \frac{d\ sc(y^*)}{dy}$$

- that is, price should equal short-run marginal cost. Suppose this is true at some y^1.
- Let $tvc(y)$ denote the total variable cost, and let tfc denote the total fixed cost. Then

$$\pi^1 = py^1 - tvc(y^1) - tfc$$

Optimal Shutdown

- If π^1 is negative, the firm is better off shutting down and producing nothing ($y = 0$). Let π^0 denote profits when $y = 0$:

$$\pi^0 = p \cdot 0 - tvc(0) - tfc = -tfc < 0$$

- The firm will produce $y^1 > 0$ only if $\pi^1 \geq \pi^0$, or

$$py^1 - tvc(y^1) \geq 0$$

$$p \geq \frac{tvc(y^1)}{y^1} = avc(y^1)$$

- Thus, the firm will shut down if the output price p is less than the average variable cost of y^1.

Example: Cobb-Douglas

- Let's work through the properties of the general Cobb-Douglas production function $f(x_1, x_2) = x_1^{\alpha_1} x_2^{\alpha_2}$
- $\alpha_1, \alpha_2 > 0$, but do not necessarily add up to 1.
- $f(x_1, x_2)$ is homogeneous of degree $\alpha_1 + \alpha_2$, which determines the returns to scale.
- If $\alpha_1 + \alpha_2$ is less than/equal to/greater than 1, returns to scale are decreasing/constant/increasing.
- The MRTS of x_1 for x_2 is:

$$MRTS_{1,2} = \frac{\partial f / \partial x_1}{\partial f / \partial x_2} = \frac{\alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2}}{\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2 - 1}} = \frac{\alpha_1 x_2}{\alpha_2 x_1}$$

- Since the MRTS is unchanged if $\frac{x_2}{x_1}$ is multiplied by a constant, $f(x_1, x_2)$ is homothetic.

Elasticity of Substitution

- The elasticity of substitution of x_2 for x_1 is:

$$\sigma_{21} = \frac{d \ln \left(\frac{x_2}{x_1} \right)}{d \ln MRTS_{1,2}} = \frac{d \ln \left(\frac{x_2}{x_1} \right)}{d \ln (\frac{\alpha_1 x_2}{\alpha_2 x_1})}$$

$$= \frac{\frac{x_1}{x_2}}{\frac{\alpha_1}{\alpha_2} (\frac{\alpha_2}{\alpha_1} \frac{x_1}{x_2})} = 1$$

- Like CES, the elasticity of substitution is a constant.
- If the ratio of inputs x_2/x_1 increases by 1%, the MRTS also increases by 1%.

Cost Function

- Given input prices w_1, w_2 and output level y, the cost function is

$$c(w_1, w_2, y) = \min_{x_1, x_2} w_1 x_1 + w_2 x_2 \qquad \text{s.t.} \qquad x_1^{\alpha_1} x_2^{\alpha_2} \geq y$$

- Form the Lagrangian and find the FOC:

$$L(w_1, w_2, \lambda) = w_1 x_1 + w_2 x_2 - \lambda(x_1^{\alpha_1} x_2^{\alpha_2} - y)$$

$$\frac{\partial L}{\partial x_1} = w_1 - \lambda \alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2} = 0$$

$$\frac{\partial L}{\partial x_2} = w_2 - \lambda \alpha_2 x_1^{\alpha_1} x_2^{\alpha_2 - 1} = 0$$

$$\frac{\partial L}{\partial \lambda} = x_1^{\alpha_1} x_2^{\alpha_2} \geq y = 0$$

- The first two equations give the usual optimality condition MRTS=w_1/w_2:

$$\begin{aligned}
MRTS &= \frac{\alpha_1 x_2}{\alpha_2 x_1} = \frac{w_1}{w_2} \Rightarrow \frac{w_1 x_1}{w_2 x_2} = \frac{\alpha_1}{\alpha_2} \\
x_1 &= \frac{\alpha_1 w_2 x_2}{\alpha_2 w_1} \\
y &= \left(\frac{\alpha_1 w_2 x_2}{\alpha_2 w_1}\right)^{\alpha_1} x_2^{\alpha_2} = \left(\frac{\alpha_1 w_2}{\alpha_2 w_1}\right)^{\alpha_1} x_2^{\alpha_1+\alpha_2} \\
x_2^* &= \left(y\left(\frac{\alpha_1 w_2}{\alpha_2 w_1}\right)^{-\alpha_1}\right)^{\frac{1}{\alpha_1+\alpha_2}} \\
x_1^* &= \left(y\left(\frac{\alpha_2 w_1}{\alpha_1 w_2}\right)^{-\alpha_2}\right)^{\frac{1}{\alpha_1+\alpha_2}}
\end{aligned}$$

- The conditional input demand functions are given by:

$$x_1^* = \left(y \left(\frac{\alpha_2 w_1}{\alpha_1 w_2} \right)^{-\alpha_2} \right)^{\frac{1}{\alpha_1+\alpha_2}}, \qquad x_2^* = \left(y \left(\frac{\alpha_1 w_2}{\alpha_2 w_1} \right)^{-\alpha_1} \right)^{\frac{1}{\alpha_1+\alpha_2}}$$

- The cost function is:

$$\begin{aligned} c(w_1, w_2, y) &= w_1 x_1^* + w_2 x_2^* \\ &= w_1 \left(y \left(\frac{\alpha_2 w_1}{\alpha_1 w_2} \right)^{-\alpha_2} \right)^{\frac{1}{\alpha_1+\alpha_2}} + w_2 \left(y \left(\frac{\alpha_1 w_2}{\alpha_2 w_1} \right)^{-\alpha_1} \right)^{\frac{1}{\alpha_1+\alpha_2}} \end{aligned}$$

- If $\alpha_1 + \alpha_2 = 1$, then this becomes

$$c(w_1, w_2, y) = y \left(w_1 \left(\frac{\alpha_2 w_1}{\alpha_1 w_2} \right)^{-\alpha_2} + w_2 \left(\frac{\alpha_1 w_2}{\alpha_2 w_1} \right)^{-\alpha_1} \right)$$

Short-Run Cost

- Suppose that x_1 is fixed to be $\bar{x}_1$, and only x_2 can be changed.
- The short-run cost function is:

$$\min_{x_2} w_1 \bar{x}_1 + w_2 x_2 \qquad s.t. \qquad \bar{x}_1^{\alpha_1} x_2^{\alpha_2} \geq y$$

- Since $f(x_1, x_2)$ is strictly increasing, the optimal x_2 will simply be the value that produces exactly y:

$$x_2^* = \left(\frac{y}{\bar{x}_1^{\alpha_1}} \right)^{\frac{1}{\alpha_2}}$$

- The short-run cost function is then:

$$SRC(w_1, w_2, y) = w_1 \bar{x}_1 + w_2 \left(\frac{y}{\bar{x}_1^{\alpha_1}} \right)^{\frac{1}{\alpha_2}}$$

- The first term is the *fixed cost*, and the second term is the *variable cost*

- Suppose $\alpha_1 + \alpha_2 = 1$ (constant returns to scale). The long-run cost function is:

$$c(w_1, w_2, y) = y\left(w_1\left(\frac{\alpha_2 w_1}{\alpha_1 w_2}\right)^{-\alpha_2} + w_2\left(\frac{\alpha_1 w_2}{\alpha_2 w_1}\right)^{-\alpha_1}\right)$$

- Long-run marginal cost is the derivative with respect to y, which is simply:

$$LRMC = \left(w_1\left(\frac{\alpha_2 w_1}{\alpha_1 w_2}\right)^{-\alpha_2} + w_2\left(\frac{\alpha_1 w_2}{\alpha_2 w_1}\right)^{-\alpha_1}\right)$$

- This is also equal to long-run average cost, which is just $c(w_1, w_2, y)/y$.
- These properties hold for any production function with constant returns to scale (Exercise 3.36 and 3.37).

Profit Function

- The profit function is the optimal value of the problem:

$$\pi(p, w_1, w_2) = \max_{x_1, x_2} py - w_1 x_1 - w_2 x_2 \qquad s.t. \qquad x_1^{\alpha_1} x_2^{\alpha_2} \geq y$$

- We can substitute in $x_1^{\alpha_1} x_2^{\alpha_2}$ for y, eliminating the constraint and giving us an unconstrained problem:

$$\pi(p, w_1, w_2) = \max_{x_1, x_2} p x_1^{\alpha_1} x_2^{\alpha_2} - w_1 x_1 - w_2 x_2$$

- Assume that $\alpha_1 + \alpha_2 \leq 1$. $f(x_1, x_2)$ is homogeneous of degree $\alpha_1 + \alpha_2$; we saw last week that this implies it is concave.
- Therefore, if we find an inflection point of the objective function, it is the maximum.

Profit Function

$$\pi(p, w_1, w_2) = \max_{x_1, x_2} \; p x_1^{\alpha_1} x_2^{\alpha_2} - w_1 x_1 - w_2 x_2$$

- The FOC are:

$$\frac{\partial}{\partial x_1} = p\alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2} - w_1 = 0$$
$$\frac{\partial}{\partial x_2} = p\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2 - 1} - w_2 = 0$$

Profit Function

- We can rewrite the FOC to get the marginal revenue product $=$ marginal cost optimality condition:

$$p\alpha_1 x_1^{\alpha_1 - 1} x_2^{\alpha_2} = w_1 \quad \Rightarrow \quad x_1 = p\alpha_1 \frac{y}{w_1}$$

$$p\alpha_2 x_1^{\alpha_1} x_2^{\alpha_2 - 1} = w_2 \quad \Rightarrow \quad x_2 = p\alpha_2 \frac{y}{w_2}$$

- We want to find the profit function, input demand function, and output supply function. Let x_1^*, x_2^*, y^* denote the optimal values of x_1, x_2, y. Then:

$$y^* = (x_1^*)^{\alpha_1} (x_2^*)^{\alpha_2} = \left(p\alpha_1 \frac{y^*}{x_1}\right)^{\alpha_1} \left(p\alpha_2 \frac{y^*}{x_2}\right)^{\alpha_2}$$

$$(y^*)^{1-\alpha_1-\alpha_2} = \left(\frac{p\alpha_1}{x_1}\right)^{\alpha_1} \left(\frac{p\alpha_2}{x_2}\right)^{\alpha_2}$$

Profit Function

- Output supply, input demand, and profit functions are given by:

$$y^* = \left(\frac{p\alpha_1}{x_1}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{p\alpha_2}{x_2}\right)^{\frac{\alpha_2}{1-\alpha_1-\alpha_2}}$$

$$x_1^* = p\alpha_1 \frac{y^*}{w_1}, \qquad \boldsymbol{x}_2^* = p\alpha_2 \frac{y^*}{w_2}$$

$$\pi(p, w_1, w_2) = py^* - w_1 x_1^* - w_2 x_2^*$$

- If we knew the profit function, then Hotelling's lemma states that we can find the output supply and input demand functions by differentiating:

$$\frac{\partial \pi}{\partial p} = y^*, \qquad \frac{\partial \pi}{\partial w_i} = x_i^*$$

Chapter 4: Partial Equilibrium

- So far, we've covered the decision problems of consumers and firms separately.
- Now, we'll combine them and examine the outcome.
- First, let's assume that markets are *perfectly competitive* for both buyers and sellers.
- Each buyer or seller is insignificant compared to the size of the entire market, therefore a *price-taker*: his actions do not affect the market price.
- An individual buyer's demand for a good is the outcome of the utility maximization problem, subject to a budget constraint.
- An individual seller's supply of a good is the outcome of the profit maximization problem, subject to technological constraints, output price, and input prices.

Market Demand

- Assume all buyers can be indexed by $i \in \{1, ..., I\}$.
- Let $q^i(p, \boldsymbol{p}, y^i)$ be buyer i's demand for good q, as a function of:
  - own price, p
  - prices of all other goods, $\boldsymbol{p}$
  - buyer i's income, y^i
- *Market demand* for good q is the sum of all individual demands:

$$q^d(p) = \sum_{i \in \{1,...,I\}} q^i(p, \boldsymbol{p}, y^i)$$

- Note that market demand depends not only on *aggregate income* of all buyers, but also the *distribution* of income among buyers.
- Since individual demand is homogenous of degree zero in $(\boldsymbol{p}, y)$, market demand is homogeneous of degree zero in $(p, \boldsymbol{p}, y^1, ..., y^I)$.

Market Supply

- We will distinguish between firms that are potential sellers in the *short run* (where some inputs are fixed) and in the *long run* (where all inputs are variable).
- Assume that in the short run, the number of potential sellers is *fixed*, and can be indexed by $j \in \{1, ..., J\}$.
- The *short-run market supply* function is the sum of individual firm short-run supply functions $q^j(p, \boldsymbol{w})$

$$q^s(p) = \sum_{j \in \{1,...,J\}} q^j(p, \boldsymbol{w})$$

- where p is the price of good q, and $\boldsymbol{w}$ are input prices.

Short-Run Equilibrium

- Together, market demand and market supply will determine the price and the total quantity traded.
- We say that a competitive market is in *short-run equilibrium* at price p^* when $q^d(p^*) = q^s(p^*)$.
- Geometrically, this corresponds to the usual intersection of the supply curve and the demand curve.
- At equilibrium, each buyer is buying his optimal amount of the good at the market price.
- Each seller is selling his profit-maximizing output at the market price.
- Therefore, no agent has any incentive to change his behavior.

Example 4.1

- Suppose the supply side is a competitive industry with J identical firms.
- Production is Cobb-Douglas: $q = x^{\alpha} k^{1-\alpha}, 0 < \alpha < 1$
- x is a variable input (e.g. labor)
- k is an input that is fixed in the short run (e.g. plant size)
- Note this technology has constant returns to scale.
- Let's derive the short-run profit and supply functions.
- w_x, w_k are input prices.

Short-Run Firm Behavior

$$\max_{q,x} pq - w_x x - w_k k \qquad \text{s.t. } x^\alpha k^{1-\alpha} = q$$

$$= \max_{x} p x^\alpha k^{1-\alpha} - w_x x - w_k k$$

- First-order condition:

$$\alpha p x^{\alpha-1} k^{1-\alpha} - w_x = 0$$

$$x^* = p^{\frac{1}{1-\alpha}} x^{\frac{1}{\alpha-1}} \alpha^{\frac{1}{1-\alpha}} k$$

$$\pi(p, w_x, w_k, k) = p^{\frac{1}{1-\alpha}} w_x^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) k - w_k k$$

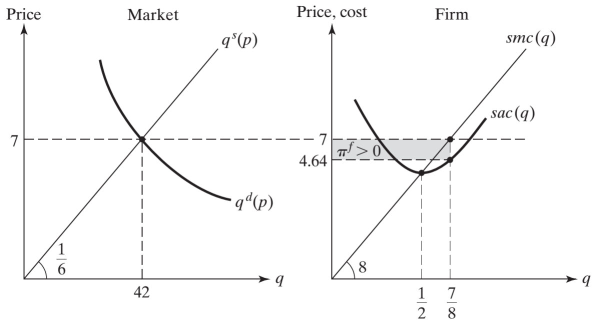
- By Hotelling's lemma, differentiating profit with respect to p gives short-run supply:

$$q(p, w_x, w_k, k) = p^{\frac{\alpha}{1-\alpha}} w_x^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{\alpha}{1-\alpha}} k$$

Short-Run Equilibrium

- Assume $\alpha = \frac{1}{2}, w_x = 4, w_k = 1, k = 1$.
- Firm supply is $q^j = \frac{p}{8}$
- Assume $J = 48$. Then market supply is $q^s = 48\frac{p}{8} = 6p$.
- Assume market demand is given by: $q^d = 294/p$.
- At short-run equilibrium: $p^* = 7, q^* = 42, q^j = 7/8, \pi^j = 2.0625$.

Short-Run Equilibrium



- Note that each firm makes a positive profit.

Long-Run Equilibrium

- In the *long run*, all inputs are variable.
- Also, firms may enter or exit the industry. Therefore, the total number of firms $\hat{J}$ is endogenous.
- Firms will enter the industry if long-run profits are positive, and exit if they are negative.
- In long-run equilibrium, two conditions must be satisfied.
- First, the market clears (supply = demand).

$$q^d(\hat{p}) = \sum_{j=1}^{\hat{J}} q^j(\hat{p})$$

- Second, long-run profits for all firms are zero, so there is no entry or exit.

$$\pi^j(\hat{p}) = 0 \qquad \text{for } j = 1, \ldots, \hat{J}$$

Example 4.2

- Suppose inverse market demand is $p = 39 - 0.009q$.
- Assume all firms have identical technology and face identical input prices.
- Long-run profit function for a representative firm is $\pi^j(p) = p^2 - 2p - 399$.
- By Hotelling's lemma, output supply is $y^j = 2p - 2$.
- Conditions that must be satisfied for a long-run equilibrium:

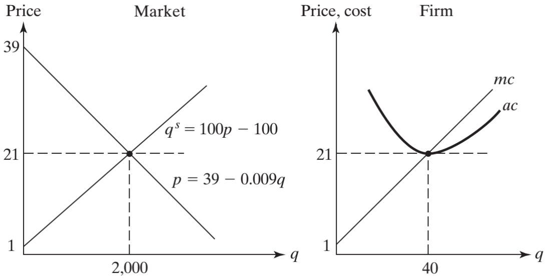
$$(1000/9)(39 - \hat{p}) = \hat{J}(2\hat{p} - 2)$$

$$\hat{p}^2 - 2\hat{p} - 399 = 0$$

$$\hat{p} = 21, \hat{J} = 50$$

- Each firm produces 40 units.

Long-Run Equilibrium



- By assumption, long-run profits are zero.

Example 4.3

- Again, suppose that production is Cobb-Douglas: $q = x^{\alpha} k^{1-\alpha}$. Now, all inputs are variable.
- Assume $\alpha = \frac{1}{2}, w_x = 4, w_k = 1, k = 1$.
- Short-run profits and supply were:

$$\pi^j(p, k) = \frac{p^2 k}{16} - k, q^j = \frac{pk}{8}$$

- Note that the optimal long-run values for profit and supply can be found by maximizing the short-run function over k.
- To satisfy the long-run zero profit condition, $\hat{p}$ must be 4. k can take any value.
- The market-clearing condition becomes:

$$q^d(\hat{p}) = \frac{294}{4} = \frac{4\hat{J}\hat{k}}{8} = q^s(\hat{p})$$

$$147 = \hat{J}\hat{k}$$

Example 4.3

$$q^d(\hat{p}) = \frac{294}{4} = \frac{4\hat{J}\hat{k}}{8} = q^s(\hat{p})$$

$$147 = \hat{J}\hat{k}$$

- There are infinitely many combinations of $\hat{J}, \hat{k}$ that satisfy this condition.
- We say the long-run equilibrium number of firms and plant size is *indeterminate*. This is true in general when technology has constant returns to scale.

Imperfect Competition

- Now, let's consider cases where firms have market power, i.e. they can affect the market price through their actions.
- The extreme case of market power is *pure monopoly*, where there is a single seller of a product.
- There are no close substitutes for the product, and entry into the industry is blocked.
- The monopolist takes the market demand function as given, and chooses price and quantity to maximize profit.
- This is equivalent to choosing quantity q, and charging exactly the inverse demand $p(q)$.

Imperfect Competition

- Let $r(q) = p(q)q$ denote revenue at output q, $c(q)$ denote costs.

$$\Pi(q) = r(q) - c(q)$$

- If the optimal choice for q^* is positive, then the first-order condition must be satisfied:

$$\Pi'(q) = r'(q^*) - c'(q^*) = 0$$

Pure Monopoly

$$\Pi'(q) = r'(q^*) - c'(q^*) = 0$$

- This is the usual requirement that marginal revenue = marginal cost:

$$MR(q^*) = MC(q^*)$$

- Differentiate $r(q) = p(q)q$ to get MR:

$$MR(q) = p(q) + q\frac{dp(q)}{dq} = p(q)\left[1 + \frac{dp(q)}{dq}\frac{q}{p(q)}\right]$$

$$= p(q)\left[1 + \frac{1}{\epsilon(q)}\right]$$

- where $\epsilon(q)$ is the elasticity of market demand at output q:

$$\epsilon(q) = \frac{dq}{dp}\frac{p}{q}$$

Pure Monopoly

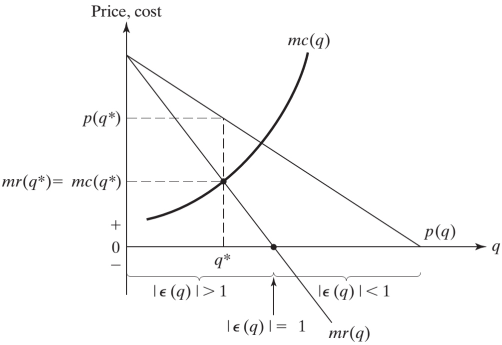
$$\Pi'(q) = r'(q^*) - c'(q^*) = 0$$

- We assume that $\epsilon(q) < 0$, therefore market demand is downward sloping.

$$p(q^*)\left[1 + \frac{1}{\epsilon(q^*)}\right] = MC(q^*) \geq 0$$

- By assumption, MC and price are always non-negative.
- Therefore, $|\epsilon(q)| \geq 1$: the monopolist never chooses output in the *inelastic* (i.e. elasticity < 1) range of market demand.

Pure Monopoly



- In equilibrium, the deviation of price from MC is:

$$\frac{p(q^*) - MC(q^*)}{p(q^*)} = \frac{1}{|\epsilon(q^*)|}$$

- The more inelastic market demand, the greater the deviation from $p = MC$.

Equilibrium and Welfare

- So far, we have been analyzing different market structures and predicting the equilibrium price and quantity traded, assuming that consumers and producers behave rationally.
- This is a *positive* question: we are simply concerned about making a prediction, without saying whether it is socially desirable.
- Now, we will ask if these outcomes are preferable from a *social* point of view.
- This is a *normative* question: we are asking whether an outcome is more beneficial to society.
- We will need to define what "welfare" is, and how it is affected by changes in prices and quantities.

Partial Equilibrium Approach

- We want to see what the effect of a change in prices and quantities of a certain good q has on a person's welfare.
- Assume that the price of every other good except q remains fixed. This is *partial equilibrium* analysis.
- Let p denote the price of good q; $\boldsymbol{p}$ denotes the price of *all other goods*.
- Indirect utility $= v(p, \boldsymbol{p}, y)$. We will sometimes just write $v(p, y)$.
- Let m be the amount of income spent on all other goods than q. This is a *composite commodity* that represents the "quantity" of all other goods.
- If $\boldsymbol{x}(p, \boldsymbol{p}, y)$ is demand for all other goods, then

$$m(p, \boldsymbol{p}, y) = \boldsymbol{p} \cdot \boldsymbol{x}(p, \boldsymbol{p}, y)$$

Exercise 4.16

- Let $u(q, \mathbf{x})$ denote the consumer's utility function over all goods.
- Under the usual assumptions on utility functions, then the 2-good utility function $\overline{u}(q, m)$ defined by:

$$\overline{u}(q, m) = \max_{\mathbf{x}} u(q, \mathbf{x}) \qquad \text{s.t. } \mathbf{p} \cdot \mathbf{x} \leq m$$

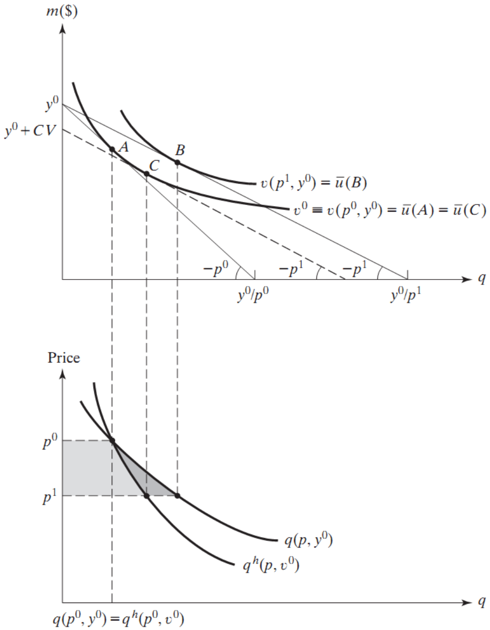
- also satisfies these assumptions, and the derived indirect utility and demand functions from this problem match the original utility function.

Compensating Variation

- Suppose we are evaluating a policy that will result in a *decrease* in price of good q.
- What is the consumer's *willingness to pay* for this price decrease? We can determine this if we know the consumer's demand curve.
- Suppose the consumer's income is y^0.
- The initial price of the good is p^0. As a result of the policy, it will decrease to p^1.
- The consumer's utility *before* the price change is $v(p^0, y^0)$; after, it is $v(p^1, y^0)$.
- The amount of income CV the consumer is willing to give up for the price decrease must satisfy:

$$v(p^1, y^0 + CV) = v(p^0, y^0)$$

- CV stands for *compensating variation*.



m($)
y0
y0 + CV
A
B
C
v(p1, y0) = ū(B)
v0 ≡ v(p0, y0) = ū(A) = ū(C)
−p0
−p1
−p1
y0/p0
y0/p1
q
Price
p0
p1
q(p, y0)
qh(p, v0)
q
q(p0, y0) = qh(p0, v0)

Compensating Variation

- Using the relationship between expenditure and indirect utility:

$$e(p^1, v(p^0, y^0)) = e(p^1, v(p^1, y^0 + CV)) = y^0 + CV$$

- using $y^0 = e(p^0, v(p^0, y^0))$ and let $v^0 = v(p^0, y^0)$:

$$CV = e(p^1, v^0) - e(p^0, v^0)$$

- By Shephard's lemma, $\frac{\partial e}{\partial p} = q^h(p, y)$:

$$CV = \int_{p^0}^{p^1} \frac{\partial e(p, v^0)}{\partial p} dp = \int_{p^0}^{p^1} q^h(p, v^0) dp$$

- Therefore, CV is the area to the *left* of the Hicksian demand curve from p^0 to p^1.
- If $p^1 < p^0$, CV is the negative of the area: a negative income adjustment is necessary to restore the original utility level.

Compensating Variation

- One practical problem with CV is that it is based on the Hicksian demand curve, which we cannot directly observe.
- We can observe the Marshallian demand curve, which shows the total effect of a price change (substitution effect + income effect). CV is a substitution effect.
- The Marshallian demand curve shows *consumer surplus*.
- At (p^0, y^0), the consumer surplus $CS(p^0, y^0)$ is the area under the demand curve and above the price p^0.
- The change in CS due to a price decrease from p^0 to p^1 is:

$$\Delta CS = CS(p^1, y^0) - CS(p^0, y^0) \quad \int_{p^1}^{p^0} q(p, y^0) dp$$

- ΔCS is opposite in sign to CV.

Consumer Surplus

- We want to know CV, but we can only calculate ΔCS. How good of an approximation is it?
- As long as the income effect is small compared to ΔCS, which is true if the change in price is small enough.
- Note that this is based on the demand curve for a single individual.
- If we observe a market demand curve with many individual consumers, ΔCS will give an approximation of the total amount of income consumers are willing to give up, but won't tell us how the total cost should be *distributed* among consumers.

Pareto Efficiency

- How can we judge whether a policy or project that will result in a change in prices and quantities is worth doing?
- If it possible to make *at least one person better off* while *no one becomes worse off*, we say that it is possible to make a *Pareto improvement*.
- If there is no way to make a Pareto improvement, then the situation is *Pareto efficient*: there is no change that can be made that would not make someone worse off.
- The idea of Pareto efficiency is widely used in economics to evaluate the performance of a system.
- If a system is Pareto efficient, it is not "wasting" any resources (though this concept does not address issues of distribution and inequality).

Administrative Stuff

- Homework #2 is due at the end of class. I will post the solutions on the website later today.
- The midterm will next week, on April 17.
- Midterm will be open-book.
- Chapters 1, 2.1, 2.4, 3, and 4.1-4.2 (up to monopoly) will be covered.
- Sample midterms from previous semesters are on the course website.