Advanced Microeconomic Analysis, Lecture 7

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Administrative Stuff

- The midterm exam will be returned next week.
- I will post a new homework, HW #3, on the website later today. It will be due in two weeks.
- I have decided to change the weights on the homeworks and exams for computing the final grade, to follow regulations.
- The new percentages will be: Homework - 5%, Midterm - 25%, Final Exam - 70%.

Equilibrium and Welfare

- So far, we have been analyzing different market structures and predicting the equilibrium price and quantity traded, assuming that consumers and producers behave rationally.
- This is a *positive* question: we are simply concerned about making a prediction, without saying whether it is socially desirable.
- Now, we will ask if these outcomes are preferable from a *social* point of view.
- This is a *normative* question: we are asking whether an outcome is more beneficial to society.
- We will need to define what "welfare" is, and how it is affected by changes in prices and quantities.

Partial Equilibrium Approach

- We want to see what the effect of a change in prices and quantities of a certain good q has on a person's welfare.
- Assume that the price of every other good except q remains fixed. This is *partial equilibrium* analysis.
- Let p denote the price of good q; $\boldsymbol{p}$ denotes the price of *all other goods*.
- Indirect utility $= v(p, \boldsymbol{p}, y)$. We will sometimes just write $v(p, y)$.
- Let m be the amount of income spent on all other goods than q. This is a *composite commodity* that represents the "quantity" of all other goods.
- If $\boldsymbol{x}(p, \boldsymbol{p}, y)$ is demand for all other goods, then

$$m(p, \boldsymbol{p}, y) = \boldsymbol{p} \cdot \boldsymbol{x}(p, \boldsymbol{p}, y)$$

Exercise 4.16

- Let $u(q, \boldsymbol{x})$ denote the consumer's utility function over all goods.
- Under the usual assumptions on utility functions, then the 2-good utility function $\overline{u}(q, m)$ defined by:

$$\overline{u}(q, m) = \max_{\boldsymbol{x}} u(q, \boldsymbol{x}) \qquad \text{s.t. } \boldsymbol{p} \cdot \boldsymbol{x} \leq m$$

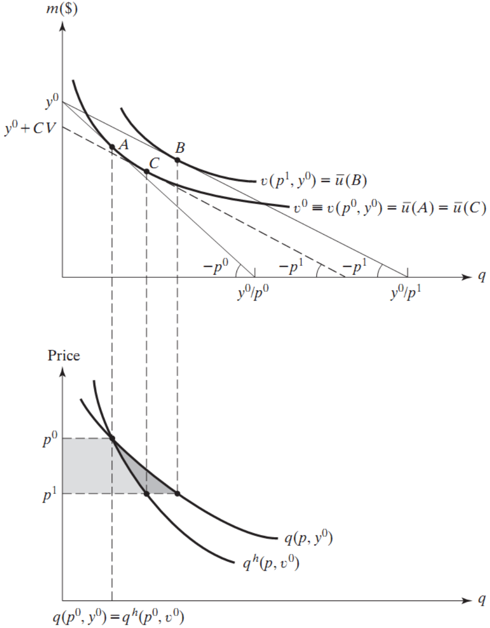
- also satisfies these assumptions, and the derived indirect utility and demand functions from this problem match the original utility function.

Compensating Variation

- Suppose we are evaluating a policy that will result in a *decrease* in price of good q.
- What is the consumer's *willingness to pay* for this price decrease? We can determine this if we know the consumer's demand curve.
- Suppose the consumer's income is y^0.
- The initial price of the good is p^0. As a result of the policy, it will decrease to p^1.
- The consumer's utility *before* the price change is $v(p^0, y^0)$; after, it is $v(p^1, y^0)$.
- The amount of income CV the consumer is willing to give up for the price decrease must satisfy:

$$v(p^1, y^0 + CV) = v(p^0, y^0)$$

- CV stands for *compensating variation*.



m($)
y^0
$y^0 + CV$
A
B
C
$v(p^1, y^0) = \bar{u}(B)$
$v^0 \equiv v(p^0, y^0) = \bar{u}(A) = \bar{u}(C)$
$-p^0$
$-p^1$
$-p^1$
y^0/p^0
y^0/p^1
q
Price
p^0
p^1
$q(p, y^0)$
$q^h(p, v^0)$
q
$q(p^0, y^0) = q^h(p^0, v^0)$

Compensating Variation

- Using the relationship between expenditure and indirect utility:

$$e(p^1, v(p^0, y^0)) = e(p^1, v(p^1, y^0 + CV)) = y^0 + CV$$

- using $y^0 = e(p^0, v(p^0, y^0))$ and let $v^0 = v(p^0, y^0)$:

$$CV = e(p^1, v^0) - e(p^0, v^0)$$

- By Shephard's lemma, $\frac{\partial e}{\partial p} = q^h(p, y)$:

$$CV = \int_{p^0}^{p^1} \frac{\partial e(p, v^0)}{\partial p} dp = \int_{p^0}^{p^1} q^h(p, v^0) dp$$

- Therefore, CV is the area to the *left* of the Hicksian demand curve from p^0 to p^1.
- If $p^1 < p^0$, CV is the negative of the area: a negative income adjustment is necessary to restore the original utility level.

Compensating Variation

- One practical problem with CV is that it is based on the Hicksian demand curve, which we cannot directly observe.
- We can observe the Marshallian demand curve, which shows the total effect of a price change (substitution effect + income effect). CV is a substitution effect.
- The Marshallian demand curve shows *consumer surplus*.
- At (p^0, y^0), the consumer surplus $CS(p^0, y^0)$ is the area under the demand curve and above the price p^0.
- The change in CS due to a price decrease from p^0 to p^1 is:

$$\Delta CS = CS(p^1, y^0) - CS(p^0, y^0) \quad \int_{p^1}^{p^0} q(p, y^0) dp$$

- ΔCS is opposite in sign to CV.

Consumer Surplus

- We want to know CV, but we can only calculate ΔCS. How good of an approximation is it?
- As long as the income effect is small compared to ΔCS, which is true if the change in price is small enough.
- Note that this is based on the demand curve for a single individual.
- If we observe a market demand curve with many individual consumers, ΔCS will give an approximation of the total amount of income consumers are willing to give up, but won't tell us how the total cost should be *distributed* among consumers.

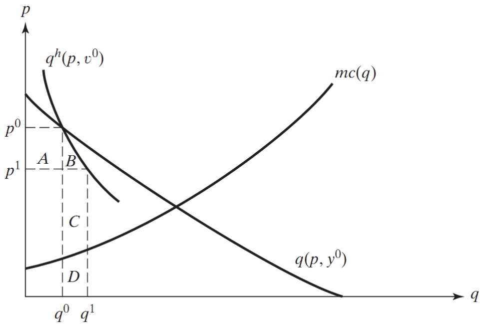
Pareto Efficiency

- How can we judge whether a policy or project that will result in a change in prices and quantities is worth doing?
- If it possible to make *at least one person better off* while *no one becomes worse off*, we say that it is possible to make a *Pareto improvement*.
- If there is no way to make a Pareto improvement, then the situation is *Pareto efficient*: there is no change that can be made that would not make someone worse off.
- The idea of Pareto efficiency is widely used in economics to evaluate the performance of a system.
- If a system is Pareto efficient, it is not "wasting" any resources (though this concept does not address issues of distribution and inequality).

Efficiency of Market Structures

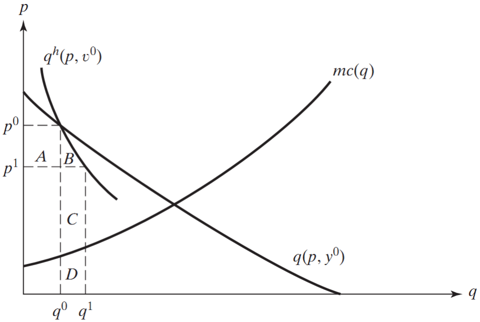
- Let's compare the types of market competition we've seen: perfect competition, and monopoly, to see if they yield a Pareto efficient outcome.
- The only difference is in prices and quantities in equilibrium.
- What values of (p, q) are Pareto-efficient outcomes?
- Suppose there is one producer and one consumer.

Monopoly



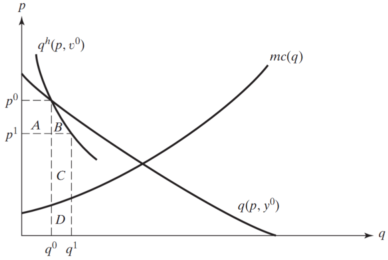
- $q(p, y^0)$ is Marshallian demand of the consumer, $q^h(p, v^0)$ is Hicksian demand at $v^0 = v(p^0, y^0)$.
- The firm's marginal cost curve is the same as the supply curve after average variable cost is minimized.
- If the firm behaved as a price-taker, equilibrium would be at intersection of curves.

Monopoly



- Consider (p^0, y^0) above the competitive point. We will show this is not a Pareto-efficient outcome.
- Suppose we reduce the price of q from p^0 to p^1.
- Consumer's willingness to pay is $CV = A + B$.

Monopoly



- Change in firm's profits is:

$$\left[p^1 q^1 - c(q^1)\right] - \left[p^0 q^0 - c(q^0)\right] = \left[p^1 q^1 - p^0 q^0\right] - \left[c(q^1) - c(q^0)\right]$$

$$= \left[p^1 q^1 - p^0 q^0\right] - \int_{q^0}^{q^1} MC(q)dq = C + D - A - D = C - A$$

- We can transfer A from consumer to producer. Consumer gains B, producer gains C, both are better off.
- Therefore, the original situation was not Pareto-efficient.

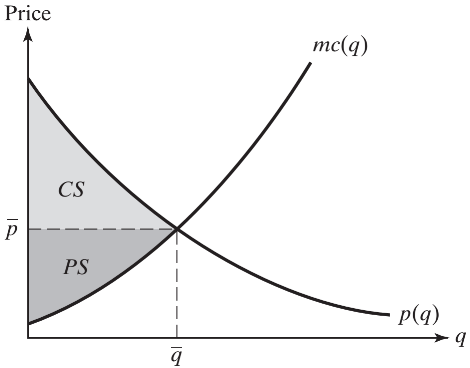
Monopoly

- The same reasoning applies to points on the Marshallian demand curve below the competitive point.
- The only price-quantity pair that can be Pareto-efficient is the perfectly competitive outcome.

Efficiency and Total Surplus Maximization

- Consumer surplus is close to being a dollar measure of the gains going to a consumer due to purchasing a good.
- For producers, we can exactly measure the gain: the *producer surplus* is simply revenues above variable costs.
- Under the assumption that demand is downward-sloping and firm's marginal costs are rising, a necessary (but not sufficient) condition for Pareto efficiency is: $CS + PS$ is maximized.

Total Surplus



$$CS + PS = \left[\int_0^q p(z)dz - p(q)q \right] + [p(q)q - TVC(q)]$$

$$= \int_0^q p(z)dz - TVC(q) = \int_0^q [p(z) - MC(z)]\, dz$$

Maximizing Total Surplus

$$CS + PS = \int_0^p [p(z) - MC(z)]\, dz$$

- Choosing q to maximize this expression gives the first-order condition
$$p(q) = MC(q)$$
- which is the perfectly competitive equilibrium.
- Whenever price and MC differ, a Pareto improvement can be implemented, as we saw earlier.
- $p = MC$ is necessary for maximizing total surplus, but we have seen that in monopoly, $p > MC$.

Summary of Consumer/Producer Behavior

- In Chapters 1-4, we have attempted to explain (and predict) consumer and producer behavior, starting from *first principles*.
- In Chapter 1, we started with preferences over bundles of goods, that satisfied some "reasonable" axioms.
- Then, we showed that the *consumer's problem* is to choose a most preferred bundle of goods, subject to his budget constraint.
- Given the usual assumptions on preferences (and therefore the utility function that represents those preferences), we are able to derive additional concepts such as:
  - Marshallian and Hicksian demand
  - Indirect utility
  - the Minimum Expenditure problem and its solution, the expenditure function

Summary of Consumer/Producer Behavior

- Each of these are related to the others; each is enough to give a complete description of the consumer's behavior.
- For example, given an expenditure function, we can reconstruct the utlility function (and vice versa).
- In Chapter 2, we applied the same tools to firms. We assumed that the essence of a firm is that it transforms inputs into outputs, with limits determined by the production function.
- The firm's problem is to choose its inputs to maximize profits.
- From this problem, we can derive:
  - the maximized profit function
  - output supply function and input demand functions
  - Cost minimization problem and its solution, the cost function
- Similar relationships between these functions allow us to reconstruct one from the others.
- Gives a complete description of the firm's behavior.

Summary of Consumer/Producer Behavior

- In Chapter 4, we combined consumers and producers into different types of market structures.
- We were able to predict the outcome of price and quantity, arising from each agent's optimizing behavior.
- We examined outcomes in partial equilibrium (i.e. equilibrium in the market for one good), assuming that the prices of all other goods were fixed.
- We were able to classify outcomes on their Pareto-efficiency; if an outcome is Pareto-efficient, all resources are being utilized without waste (though this says nothing about inequality or fairness of distribution).

Chapter 5: General Equilibrium

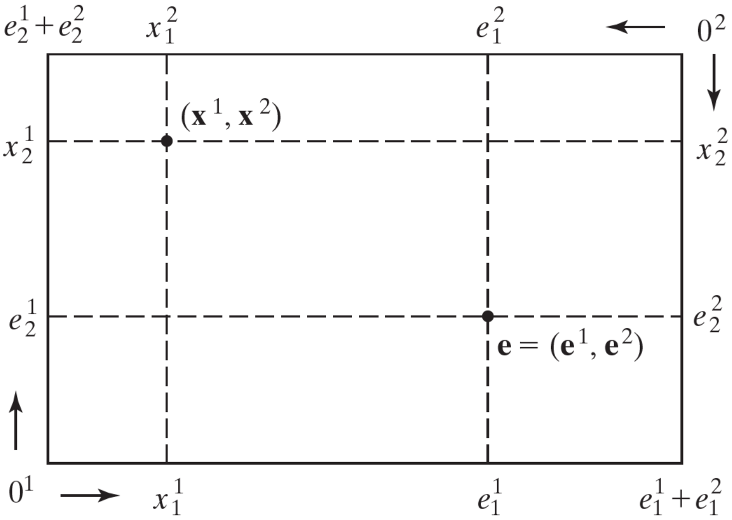
- We will cover 5.1-5.2 and 5.4 in Chapter 5.
- In Chapter 4, we considered a *partial* equilibrium (i.e. equilibrium in the market for one good).
- In this chapter, we will consider a *general* equilibrium (i.e. equilibrium in all goods, simultaneously).

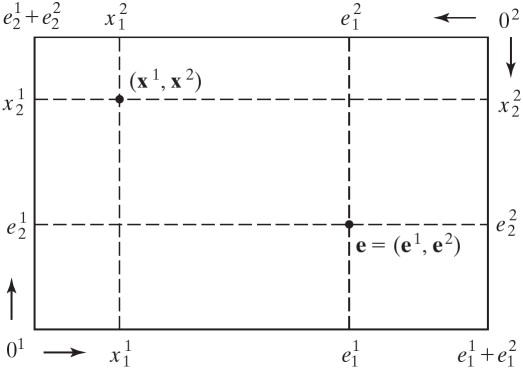
Equilibrium in Exchange

- Let's consider a very simple economy without markets (i.e. no prices, you cannot obtain goods for money).
- Instead, goods can be exchanged directly.
- Assume there are two consumers and two goods, x_1 and x_2.
- There is no production. Each consumer is *endowed* with a given quantity of each good.
- Let $\boldsymbol{e}^1 = (e_1^1, e_2^1)$ denote the *endowment* of consumer 1: he starts out with $x_1 = e_1^1, x_2 = e_2^1$.
- Likewise, $\boldsymbol{e}^2 = (e_1^2, e_2^2)$ is the endowment of consumer 2.
- The vector of the total amount of x_1 and x_2 in this economy is given by $\boldsymbol{e}^1 + \boldsymbol{e}^2$.

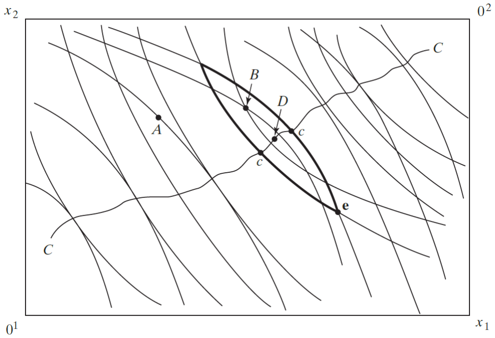
- Each consumer may consume their endowment of goods, or they can use part of it to trade, or *barter* with the other consumer.
- Consumers have a utility function $u^i(x_1, x_2)$ with the usual assumed properties.
- All trades must be voluntary.
- We want to ask the question: does a *barter equilibrium* in this economy exist, and what are its properties?
- That is, is there a sequence of voluntary trades that leads to a point where no more trade will occur?

Edgeworth Box



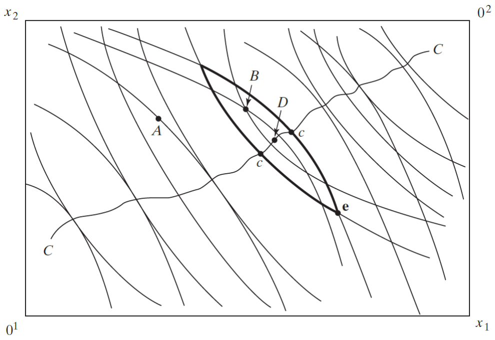


- Every point in the Edgeworth box corresponds to some division of the total endowment $\boldsymbol{e}^1 + \boldsymbol{e}^2$ among the two agents.
- For example, the point $\boldsymbol{x}^1, \boldsymbol{x}^2$ means consumer 1 has (x_1^1, x_2^1) and consumer 2 has $(e_1^1 + e_1^2 - x_1^1, e_2^1 + e_2^2 - x_2^1)$.
- The lower left corner is the origin for consumer 1, and the upper right corner is the origin for consumer 2.

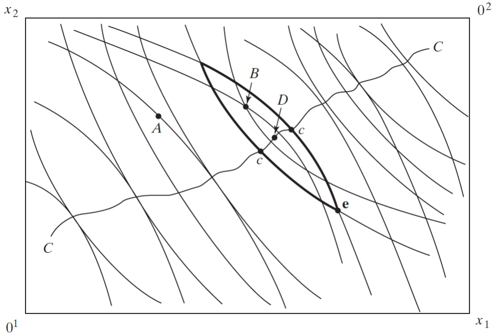


- The line *CC* is the set of allocations where the indifference curve of consumer 1 is tangent to the indifference curve of consumer 2.
- This is called the *contract curve*.

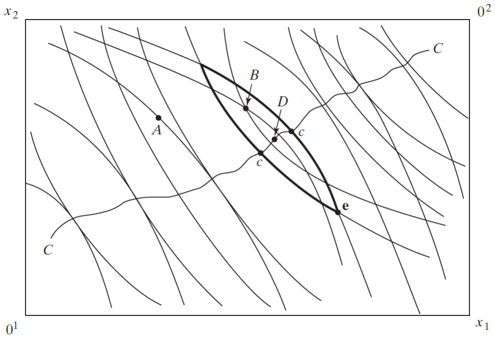
- Given an initial endowment $\boldsymbol{e}$, which allocations will be barter equilibria?
- First, only allocations in the Edgeworth box are feasible.
- Second, an allocation that makes one consumer worse off (i.e. at a lower indifference curve) will be blocked by that consumer.



- If the starting allocation is $\boldsymbol{e}$, then point A gives consumer 1 a lower utility and consumer 2 a higher utility.
- Consumer 1 will not agree to a trade going from $\boldsymbol{e}$ to A.
- Only points inside the "lens-shaped area" formed by the indifference curves going through $\boldsymbol{e}$ are agreeable to both consumers.



- Suppose the consumers trade to reach point B. The "lens" becomes smaller, and there are still possible trades that both consumers will agree to.
- Suppose the consumers reach point D. Since the indifference curves are tangent, there are no other trades that both consumers will agree to.
- D, and all points along the contract curve, are barter equilibria.



- Once such an equilibrium is reached, it is not possible to make one consumer better off, without making the other consumer worse off.
- Therefore, all barter equilibria are Pareto optimal.

Many Goods and Consumers

- Suppose there are I consumers and n goods.
- Each consumer i has a utility function $u^i(x_1, \ldots, x_n)$.
- Each consumer i is endowed with a vector $\boldsymbol{e}^i = (e^i_1, \ldots, e^i_n)$ of goods.
- The collection $(u^i, \boldsymbol{e}^i)_{i=1,\ldots,I}$ defines an *exchange economy*.
- The total endowment is $\boldsymbol{e} = \boldsymbol{e}^1 + \ldots + \boldsymbol{e}^I$.
- An *allocation* is a vector $\boldsymbol{x} = (\boldsymbol{x}^1, \ldots, \boldsymbol{x}^I)$, where $\boldsymbol{x}^i$ is consumer i's bundle of goods.
- The *set of feasible allocations* is

$$F(\boldsymbol{e}) = \left(\boldsymbol{x} \middle| \sum_i \boldsymbol{x}^i = \sum_i \boldsymbol{e}^i\right)$$

- That is, $F(\boldsymbol{e})$ is the set of all possible ways to divide the total goods among the I agents.

- In the two-consumer case, we saw that if a barter equilibrium had been reached, then no Pareto improvements were possible.
- This also holds in the general case.
- **Def. 5.1**: A feasible allocation $\boldsymbol{x} \in F(\boldsymbol{e})$ is *Pareto efficient* is there is no other feasible allocation y, such that $u^i(\boldsymbol{y}^i) \geq u^i(\boldsymbol{x}^i)$ for all consumers $i = 1, ..., I$, with at least one strict inequality.
- That is, it is not possible to make someone strictly better off without making someone else strictly worse off.
- A barter equilibrium must be Pareto-efficient, but not all Pareto-efficient allocations will be barter equilibria.
- Recall the "lens" in the 2-consumer case. Only Pareto-efficient allocations that do not give lower utility to either consumer can be barter equilibria.
- A trade to some allocation outside the "lens" would be blocked by the consumer who became worse off.
- Does this carry over to the case of many consumers?

- With many consumers, we can imagine a trade that no single consumer would want to block alone, but that a *coalition* of consumers might want to block.
- For example, suppose there are 3 consumers with $u^i(x_1, x_2, x_3) = \min(x_1, x_2, x_3)$, and the initial endowment is:

$$\boldsymbol{e}^1 = (0, 1, 1), \boldsymbol{e}^2 = (1, 0, 1), \boldsymbol{e}^3 = (1, 1, 0)$$

- Suppose a trade to $\boldsymbol{x}^1 = (1, \frac{3}{2}, \frac{3}{2}), \boldsymbol{x}^2 = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4}), \boldsymbol{x}^3 = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$ is proposed.
- Each consumer would be made better off by this trade.
- However, if consumers 2 and 3 got together, they could propose an alternative trade to $\boldsymbol{x}^1 = (0, 1, 1), \boldsymbol{x}^2 = (1, \frac{1}{2}, \frac{1}{2}), \boldsymbol{x}^3 = (1, \frac{1}{2}, \frac{1}{2})$ which makes both 2 and 3 better off compared to the previous trade.
- This is called a *blocking coalition*.

- **Def 5.2**: Let $S \in \{1, ..., I\}$ denote a coalition of consumers. We say that S blocks $\boldsymbol{x} \in F(\boldsymbol{e})$ if there is an allocation $\boldsymbol{y}$ such that:
  - $\sum_{i \in S} \boldsymbol{y}^i = \sum_{i \in S} \boldsymbol{e}^i$
  - $u^i(\boldsymbol{y}^i) \geq u^i(\boldsymbol{x}^i)$ for all $i \in S$, with at least one strict inequality.
- The conditions mean that the consumers in the coalition S can get together and divide up their total endowment among themselves, and reach a Pareto improvement for their coalition compared to $\boldsymbol{x}$.
- We say an allocation is *unblocked* if there is no coalition that can block it.
- We will require that for an allocation $\boldsymbol{x}$ to be an equilibrium, it must be unblocked.
- No consumer or group of consumers has an incentive to change the allocation by trading among themselves.
- The *core* of an exchange economy with endowment $\boldsymbol{e}$, denoted $C(\boldsymbol{e})$, is the set of all unblocked, feasible allocations.

Equlibrium in Competitive Markets

- In the pure exchange economy, there were no markets for goods or prices.
- Now, we will introduce a *decentralized* market, such that no consumer interacts with any other consumer.
- Rather, there is an impersonal market for each good with a market price, and consumers only consider the price when making their decisions.
- Equilibrium occurs when the vector of market prices is such that total demand is equal to total supply for each good, simultaneously.
- Consumers don't need to know about the preferences of other consumers, or even if they exist at all.
- Is such a price vector guaranteed to exist?

- Assume that each utility function $u^i(\cdot)$ is continuous, strongly increasing, and strictly quasiconcave.
- The consumer's income is now $\boldsymbol{p} \cdot \boldsymbol{e}^i$, the market value of his endowment.
- We can imagine the consumer selling his entire endowment at price $\boldsymbol{p}$, then using that as income for the utility maximization problem.
- Given prices $\boldsymbol{p}$, each consumer solves

$$\max_{\boldsymbol{x}^i} u^i(\boldsymbol{x}^i) \qquad \text{s.t.} \qquad \boldsymbol{p} \cdot \boldsymbol{x}^i \leq \boldsymbol{p} \cdot \boldsymbol{e}^i$$

- **Theorem 5.1**: Given our assumptions on $u^i(\cdot)$, the consumer's problem has a unique solution, $\boldsymbol{x}^i(\boldsymbol{p}, \boldsymbol{p} \cdot \boldsymbol{e}^i)$. Furthermore, this solution is continuous in $\boldsymbol{p} \in \mathbb{R}^n_{++}$ (the set of strictly positive prices).
- Market (or aggregate) demand for x_i is the sum of every consumer's demand for x_i.
- Market supply for x_i is the sum of every consumer's supply.

Excess Demand

- Aggregate excess demand for good k is denoted by:

$$z_k(\boldsymbol{p}) = \sum_i x_k^i(\boldsymbol{p}, \boldsymbol{p} \cdot \boldsymbol{e}^i) - \sum_i e_k^i$$

- Let $\boldsymbol{z} = (z_1(\boldsymbol{p}), \ldots, z_n(\boldsymbol{p}))$.
- If $z_k(\boldsymbol{p}) > 0$, the aggregate demand of good k exceeds its total endowment, and vice versa.
- Equilibrium occurs when $\boldsymbol{z}(\boldsymbol{p}) = 0$.

Properties of Aggregate Excess Demand

- **Thm 5.2** If each u^i is continuity, strongly increasing and strictly quasiconcave, then for all *strictly positive* prices $\boldsymbol{p}$:
  - Continuity: $\boldsymbol{z}(\cdot)$ is continuous at $\boldsymbol{p}$
  - Homogeneity: $\boldsymbol{z}(\lambda \boldsymbol{p}) = \boldsymbol{z}(\boldsymbol{p})$
  - Walras' Law: $p \cdot \boldsymbol{z}(\boldsymbol{p}) = 0$.
- Walras' Law says that the market value of aggregate demand must be zero at any positive price vector.
- This follows from the fact that each consumer's budget constraint is satisfied with equality.

Proof of Walras' Law

$$\max_{\mathbf{x}^i} \; u^i(\mathbf{x}^i) \qquad \text{s.t.} \qquad \mathbf{p} \cdot \mathbf{x}^i \leq \mathbf{p} \cdot \mathbf{e}^i$$

- If the budget constraint is satisfied with equality, then for consumer i:

$$\sum_{k=1}^{n} p_k x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) = \sum_{k=1}^{n} p_k e_k^i$$

$$\sum_{k=1}^{n} p_k (x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i) = 0$$

$$\sum_{i} \sum_{k=1}^{n} p_k (x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i) = 0 \qquad \text{(summing over consumers)}$$

$$\sum_{k=1}^{n} \sum_{i} p_k (x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i) = 0 \qquad \text{(reverse order of summation)}$$

$$\sum_{k=1}^{n} p_k \left(\sum_{i} x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \sum_{i} e_k^i \right) = 0$$

- The term inside the parenthesis is aggregate excess demand.

Walrasian Equilibrium

- The market for one particular good k can be in equilibrium if aggregate excess demand for good k is zero: $z_k(\boldsymbol{p}) = 0$.
- This is partial equilibrium.
- However, demand of every good can depend on the prices of every other good.
- General equilibrium is when all aggregate excess demands are 0: $\boldsymbol{z}(\boldsymbol{p}) = (0, 0, ..., 0)$
- A price vector $\boldsymbol{p}$ that equates demand and supply in every market simultaneously is called *Walrasian*.
- **Thm 5.5** If each u^i is continuous, strongly increasing, and strictly quasiconcave, then there exists at least one strictly positive $\boldsymbol{p}^*$ such that $\boldsymbol{z}(\boldsymbol{p}^*) = 0$.

- We won't go through the details of the proof of existence of Walrasian equilibrium.
- Proving it exists was one of the major achievements of mathematical economics, earning Nobel prizes for Kenneth Arrow and Gerard Debreu.
- The basic idea is to form a convex, closed and bounded set from the set of relative prices, denoted S
- Then, given some initial price vector $\boldsymbol{p}$, calculate aggregate excess demand.
- Generate a new price vector $\boldsymbol{p}'$ based on demand. If there is positive demand for good k, increase the price good k, and vice versa.
- This defines a continuous mapping from S to itself.
- Apply Brouwer's fixed point theorem, which states that under the given conditions, a fixed point of the mapping must exist.
- A fixed point, where $\boldsymbol{p} = \boldsymbol{p}'$, must satisfy $\boldsymbol{z}(\boldsymbol{p}) = 0$.

Example 5.1

- Suppose there are two goods and two consumers with CES utility:

$$u^i(x_1, x_2) = x_1^\rho + x_2^\rho, \qquad i = 1, 2,\ 0 < \rho < 1$$

- Initial endowments are: $\boldsymbol{e}^1 = (1, 0), \boldsymbol{e}^2 = (0, 1)$ (each consumer owns all of one good, none of the other).
- CES utility is strongly increasing and strictly quasiconcave for $0 < \rho < q$, so conditions for Theorem 5.5 are satisfied.
- We know CES demand of consumer i for good j is (where $r = \rho/(\rho - 1)$):

$$x_j^i(p_1, p_2, y^i) = \frac{p_j^{r-1} y^i}{p_1^r + p_2^r}$$

- Each consumer's income is: $y^1 = \boldsymbol{p} \cdot \boldsymbol{e}^1 = p_1, y^2 = \boldsymbol{p} \cdot \boldsymbol{e}^2 = p_2$.
- Since only *relative* prices matter, let's divide prices by p_2.

$$\bar{\boldsymbol{p}} = \boldsymbol{p}/p_2, \bar{p}_1 = p_1/p_2, \bar{p}_2 = 1$$

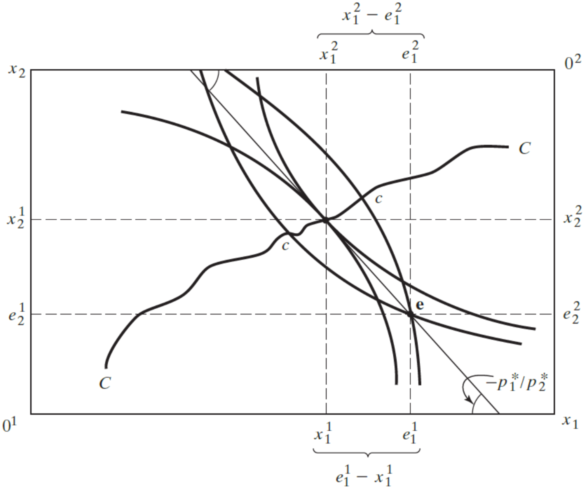
- Demand is the same at $\bar{\boldsymbol{p}}$ as it is at $\boldsymbol{p}$.

$$x_j^i(p_1, p_2, y^i) = \frac{p_j^{r-1} y^i}{p_1^r + p_2^r}$$

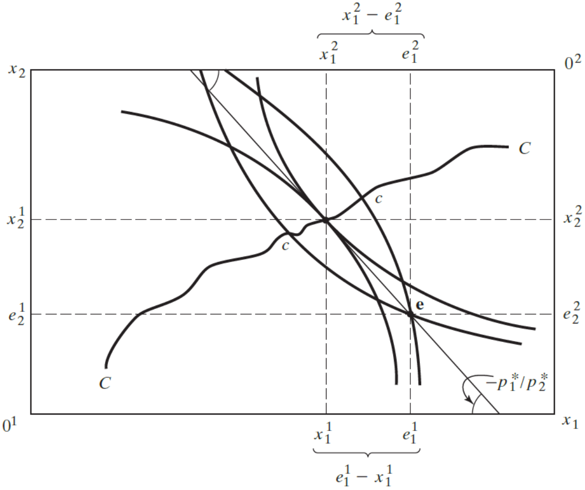
- Consider the market for good 1.
- If price vector $\bar{\boldsymbol{p}}^*$ is an equilibrium, then total demand must equal total supply:

$$x_1^1(\bar{\boldsymbol{p}}^*, \bar{\boldsymbol{p}}^* \cdot \boldsymbol{e}^1 + x_2^1(\bar{\boldsymbol{p}}^*, \bar{\boldsymbol{p}}^* \cdot \boldsymbol{e}^2 = e_1^1 + e_1^2$$
$$\frac{(\bar{p}_1^*)^{r-1} \bar{p}_1^*}{(\bar{p}_1^*)^r + 1} + \frac{(\bar{p}_1^*)^{r-1}}{(\bar{p}_1^*)^r + 1} = 1$$
$$\bar{p}_1^* = 1, \bar{p}_2^* = 1 \qquad \Rightarrow \qquad p_1^* = p_2^*$$

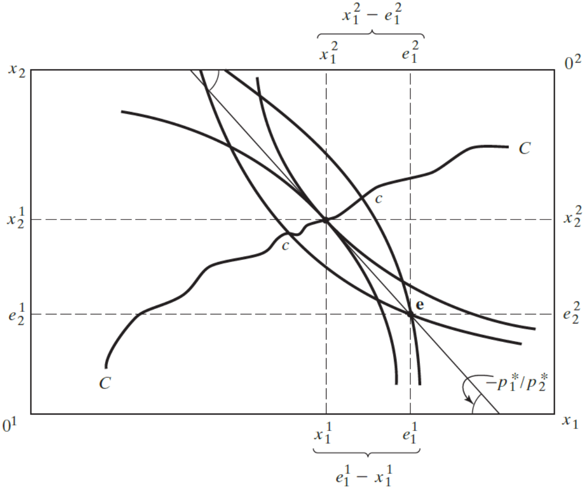
- Any price vector where $p_1^* = p_2^*$ will equate supply and demand in the market for good 1.
- By Walras' Law, $p_1^* z_1(\boldsymbol{p}^*) + p_2^* z_2(\boldsymbol{p}) = 0$.
- Since aggregate excess demand in market 1 is 0, $z_1(\boldsymbol{p}^*) = 0$, therefore $z_2(\boldsymbol{p})$ must also be 0.
- Therefore, $z(\boldsymbol{p}^*) = 0$ and $\boldsymbol{p}^*$ is a Walrasian equilibrium.



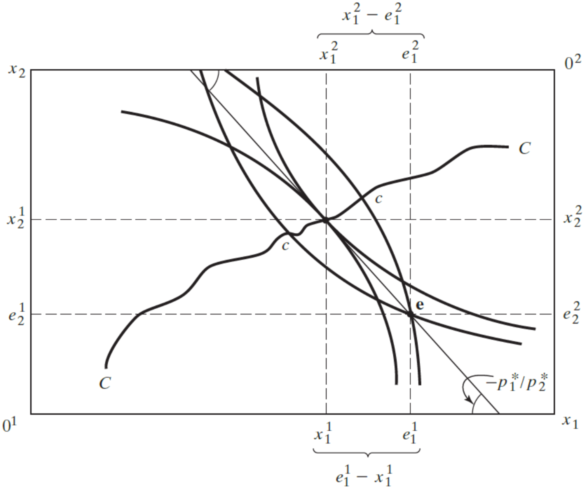
- Suppose we have a 2-good, 2-consumer economy as shown.
- Initial endowments are (e_1^1, e_2^1) and (e_1^2, e_2^2).
- At relative prices p_1^*/p_2^*, the budget line passes through point $\boldsymbol{e}$.



- Consumer 1's optimal choice, facing p_1^*/p_2^* and with income $p_1^* e_1^1 + p_2^* e_2^1$, is (x_1^1, x_2^1).
- Likewise, consumer 2's optimal choice, facing p_1^*/p_2^* and with income $p_1^* e_1^2 + p_2^* e_2^2$, is (x_1^2, x_2^2).



- Equilibrium for good 1 requires $x_1^1 + x_1^2 = e_1^1 + e_1^2$, or equivalently, $x_1^2 - e_1^2 = e_1^1 - x_1^1$.
- $x_1^2 - e_1^2$ is consumer 2's *net* demand for good 1, $e_1^1 - x_1^1$ is consumer 1's *net* supply for good 1.



- At the equilibrium point, both consumers' indifference curves are tangent to one another and the price line as well.
- Both consumers only see the market price p_1^*, p_2^* and maximize their utility; p_1^*, p_2^* equates supply and demand for all goods.

- Suppose $\boldsymbol{p}^*$ is a Walrasian equilibrium (i.e. a price vector that balances supply and demand for all goods simultaneously).
- Then the allocation of goods at this price, $\boldsymbol{x}(\boldsymbol{p}^*)$, is called a Walrasian Equilibrium Allocation (WEA).

$$\boldsymbol{x}(\boldsymbol{p}^*) = (\boldsymbol{x}^1(\boldsymbol{p}^*, \boldsymbol{p}^* \cdot \boldsymbol{e}^1), \ldots, \boldsymbol{x}^I(\boldsymbol{p}^*, \boldsymbol{p}^* \cdot \boldsymbol{e}^I))$$

- **Lemma 5.1**: $\boldsymbol{x}(\boldsymbol{p}^*) \in F(\boldsymbol{e})$, that is, it must be feasible.
- **Lemma 5.2**: Suppose that $u^i(\cdot)$ is strictly increasing, and that consumer i's demand is well defined at $\boldsymbol{p} \geq 0$ and equal to $\hat{\boldsymbol{x}}^i$. Let $\boldsymbol{x}^i$ be some other vector of goods. Then:
  - If $u^i(\boldsymbol{x}^i) > u^i(\hat{\boldsymbol{x}}^i)$, then $\boldsymbol{p} \cdot \boldsymbol{x}^i > \boldsymbol{p} \cdot \hat{\boldsymbol{x}}^i$.
  - If $u^i(\boldsymbol{x}^i) \geq u^i(\hat{\boldsymbol{x}}^i)$, then $\boldsymbol{p} \cdot \boldsymbol{x}^i \geq \boldsymbol{p} \cdot \hat{\boldsymbol{x}}^i$.
- That is, suppose there is some other allocation $\boldsymbol{x}^i$ that gives more utility to consumer i.
- Then it must be unaffordable at the current price $\boldsymbol{p}$ (otherwise, the consumer would choose it).

Administrative Stuff

- The midterm exam will be returned next week.
- I will post a new homework, HW #3, on the website later today. It will be due in two weeks.
- I have decided to change the weights on the homeworks and exams for computing the final grade, to follow regulations.
- The new percentages will be: Homework - 5%, Midterm - 25%, Final Exam - 70%.