Advanced Microeconomic Analysis, Lecture 8

Prof. Ronaldo CARPIO

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Administrative Stuff

- I am returning the midterm this week. Mean = 61, SD = 12, max = 78.
- Percentages of final grade: Homework - 5%, Midterm - 25%, Final Exam - 70%.
- Homework #3 is due next week.

Review of Last Week

- Previously, we studied *partial equilibrium*, when supply equals demand in a single goods market.
- Now, we study *general equilibrium*, where supply equals demand in all goods markets simultaneously.
- First, we looked at a simple case: a *pure exchange* economy:
  - There are n goods and I consumers, each with the usual type of utility function over goods
  - There is no production, markets, or prices.
  - Consumer i is *endowed* (i.e. given) a quantity of each good, denoted $\mathbf{e}^i$.
  - Therefore, the total amount of goods in the economy is the sum of all endowments $\sum_i \mathbf{e}^i$.
  - Consumers can trade with each other directly, by bartering.
- An *allocation* is a particular distribution of the total goods among consumers.

Review of Last Week

- For the 2-good, 2-consumer case, we can illustrate the economy with an Edgeworth box.
- Consumers will not agree to a trade that makes them worse off (i.e. lower utility).
- Given an initial endowment $\boldsymbol{e}$, we would like to find the set of *barter equilibria*, i.e. allocations at which there is no further trade.
- The *contract curve* is the set of allocations at which the indifference curves of each consumer are tangent to each other.
- Given $\boldsymbol{e}$, the set of possible barter equilibria is the part of the contract curve that lies above each agent's indifference curve through $\boldsymbol{e}$.
- Barter equilibria are Pareto efficient: cannot make one consumer better off without making the other consumer worse off.

Review of Last Week

- Next, we introduce decentralized competitive markets.
- Suppose there is a market price for each good. Consumers do not interact with each other, only the market price.
- Given a market price vector $\boldsymbol{p}$, consumer i's income is $\boldsymbol{p} \cdot \boldsymbol{e}^i$, the market value of his endowment.
- Each consumer solves his utility maximization problem, taking $\boldsymbol{p}$ and $y = \boldsymbol{p} \cdot \boldsymbol{e}^i$ as given.
- *Aggregate excess demand*, $\boldsymbol{z}(\boldsymbol{p})$, is total demand of each good minus total supply (from endowments).
- A *Walrasian equilibrium* is a price vector $\boldsymbol{p}^*$ that equates supply and demand in all markets, i.e. $\boldsymbol{z}(\boldsymbol{p}^*) = 0$.

Walrasian equilibria and the core

- Recall that the *core* of an exchange economy, denoted $C(\boldsymbol{e})$, is the set of all unblocked (i.e. no coalition of consumers can find a better allocation), feasible allocations.
- In a 2-good, 2-consumer exchange economy, the core is the subset of the contract curve in the "lens" between each consumer's indifference curve passing through $\boldsymbol{e}$.
- **Thm 5.6** Consider an exchange economy $(u^i, \boldsymbol{e}^i)_{i \in I}$. If each $u^i(\cdot)$ is strictly increasing on $\mathbb{R}^n_+$, then every Walrasian equilibrium allocation is in the core.

Walrasian equilibria and the core

- **Thm 5.6** Consider an exchange economy $(u^i, \mathbf{e}^i)_{i \in I}$. If each $u^i(\cdot)$ is strictly increasing on $\mathbb{R}^n_+$, then every Walrasian equilibrium allocation is in the core.
- In an exchange economy with many consumers, it might seem like a very complicated task to coordinate between all the consumers so that they make the right trades to reach an unblocked allocation (i.e. in the core).
- Now, we see that the same outcome can be achieved by a completely *decentralized* price mechanism, without a need for a central planner.
- Since every allocation in the core is Pareto-efficient, this also means that all Walrasian equilibrium allocations are Pareto-efficient.

First Welfare Theorem

- **Thm 5.7** Under the assumptions of Thm 5.6 (i.e. each $u^i(\cdot)$ is strictly increasing on $\mathbb{R}^n_+$), every Walrasian equilibrium allocation is Pareto efficient.
- This provides theoretical support for the Adam Smith's idea that the "invisible hand" will lead to efficient outcomes.
- That is, in an impersonal market economy with prices, the *self-interest* of agents will cause them to choose actions that lead to the benefit of society, without the need for a central planner.
- Note, however, that Pareto efficiency does not take into account any concept of "fairness" or "equality" in terms of how goods are distributed.
- We can also ask if the converse is true: suppose there is some Pareto-efficient allocation $\boldsymbol{x}^*$ we want the economy to reach, for whatever reasons.
- Is it possible to achieve $\boldsymbol{x}^*$ in a decentralized market economy, if we can choose the initial endowment? The answer is yes.

Second Welfare Theorem

- **Thm 5.8** Consider an exchange economy $(u^i, \mathbf{e}^i)_{i \in I}$ with positive aggregate endowment $\sum_i \mathbf{e}^i$, and with each $u^i(\cdot)$ continuous, strictly increasing, and strictly quasiconcave.
- Suppose $\bar{\mathbf{x}}$ is a Pareto-efficient allocation for this economy, and that endowments are redistributed so that the new endowment is $\bar{\mathbf{x}}$.
- Then, $\bar{\mathbf{x}}$ is a Walrasian equilibrium allocation of the resulting exchange economy $(u^i, \bar{\mathbf{x}}^i)_{i \in I}$.
- This theorem says that a system based on decentralized markets and self-interest can sustain any socially desired allocation of resources (as long as it is Pareto-efficient).

Exercise 5.11

- Consider a 2-good, 2-consumer economy with

$$u^1(x_1, x_2) = (x_1 x_2)^2,\ u^2(x_1, x_2) = \ln(x_1) + 2\ln(x_2)$$
$$\mathbf{e}^1 = (18, 4),\ \mathbf{e}^2 = (3, 6)$$

- Note that $x_1^2 = 21 - x_1^1$ and $x_2^2 = 10 - x_2^1$.
- First, let's characterize the set of Pareto-efficient allocations.
- This is the contract curve, i.e. the set of allocations where the indifference curves are tangent, or where $MRS^1 = MRS^2$.

$$MRS^1 = \frac{2x_1^1(x_2^1)^2}{2(x_1^1)^2 x_2^1} = \frac{x_2^1}{x_1^1}, \qquad MRS^2 = \frac{1/x_1^2}{2/x_2^2} = \frac{1}{2}\frac{x_2^2}{x_1^2}$$

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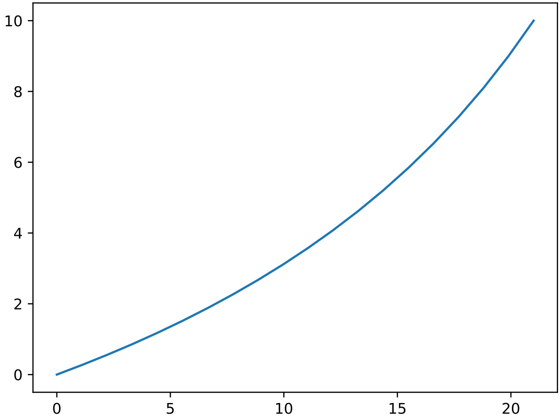
$$\frac{x_2^1}{x_1^1} = \left(\frac{1}{2}\right)\frac{10 - x_2^1}{21 - x_1^1}$$

$$\frac{10 - x_2^1}{x_2^1} = 2\frac{21 - x_1^1}{x_1^1}$$

$$\frac{10}{x_2^1} - 1 = \frac{42}{x_1^1} - 2$$

$$x_2^1 = \frac{10x_1^1}{42 - x_1^1}$$



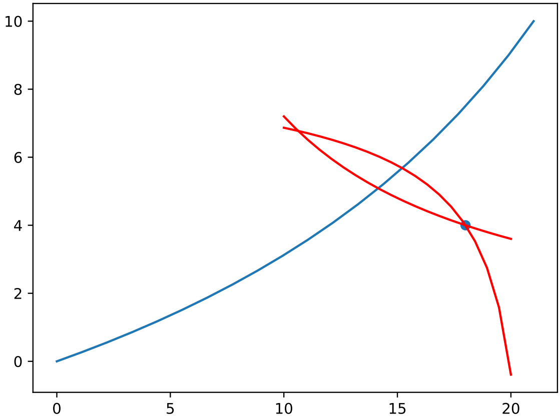
- Here we plot $x_2^1 = \frac{10x_1^1}{42-x_1^1}$ for $x_1^1 \in [0, 21]$.
- Note that the contract curve goes from $(0, 0)$ to $(21, 10)$ exactly.

- Now, let's find the core, which is the part of the contract curve that gives a higher utility for both consumers.
- Plugging in the endowments of each consumer, we get

$$u^1(18,4) = 5184, \qquad u^2(3,6) = 4.68213$$

- So we can say that the core is the part of the curve $x_2^1 = \frac{10x_1^1}{42-x_1^1}$, such that

$$(x_1^1 x_2^1)^2 \geq 5184 \qquad \text{and} \qquad \ln(21 - x_1^1) + 2\ln(10 - x_2^1) \geq 4.68213$$



- Here we plot e and the indifference curves that go through e.
- The core is the part of the contract curve between the indifference curves that go through e.

- Now, we want to find a Walrasian equilibrium.
- We want to find prices p_1, p_2 such that aggregate excess demand is zero.
- Since only relative prices matter, let's set $p_2 = 1$ and solve for p_1.
- We will need to find the Marshallian demand function for each consumer.
- To speed things up, we'll use a shortcut. Recall from Chapter 1 that an increasing transformation of a utility function represents the same preferences.
- Therefore, if we have two utility functions, $u_1(x), u_2(x)$ where $u_1(x) = f(u_2(x))$ and $f(\cdot)$ is an increasing function, then these represent the same preferences.
- Marshallian demand for and the expenditure function will be unchanged (but obviously, indirect utility will be different).

- Note that:
  - $u^1(x_1, x_2) = (x_1 x_2)^2 = (x_1^{1/2} x_2^{1/2})^4$
  - $u^2(x_1, x_2) = \ln(x_1) + 2\ln(x_2) = 3\ln\left(x_1^{1/3} x_2^{2/3}\right)$
- That is, both u^1 and u^2 are transformations of Cobb-Douglas utility functions (as you might have guessed from the MRS).
- Marshallian demand for the Cobb-Douglas utility $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$ is:

$$x_1^*(p_1, p_2, y) = \frac{\alpha y}{p_1}, \qquad , x_2^*(p_1, p_2, y) = \frac{(1-\alpha)y}{p_2}$$

- Plugging in $p_2 = 1$, $y^1 = \boldsymbol{p} \cdot \boldsymbol{e}^1 = p_1 18 + 4$, $y^2 = \boldsymbol{p} \cdot \boldsymbol{e}^2 = p_1 3 + 6$, we get

$$x_1^1 = \frac{(1/2)(p_1 18 + 4)}{p_1}, \qquad x_2^1 = \frac{(1/2)(p_1 18 + 4)}{1}$$

$$x_1^2 = \frac{(1/3)(p_1 3 + 6)}{p_1}, \qquad x_2^2 = \frac{(2/3)(p_1 3 + 6)}{1}$$

- We want to find p_1 such that $x_1^1 + x_1^2 = 18 + 3 = 21$, and $x_2^1 + x_2^2 = 6 + 4 = 10$.
- These conditions (total demand = total supply) are called *market clearing* conditions.
- We can solve each equation separately: it should give the same result. Let's try the market clearing condition for good x_2.

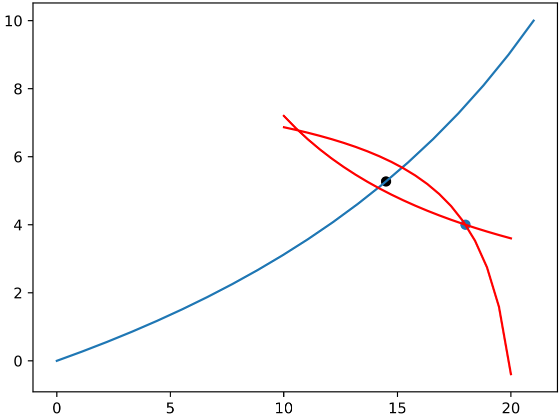
$$x_2^1 + x_2^2 = (1/2)(p_1 18 + 4) + (2/3)(p_1 3 + 6) = 10$$
$$\frac{18p_1 + 4}{2} + \frac{2(3p_1 + 6)}{3} = 10$$
$$54p_1 + 12 + 12p_1 + 24 = 60$$
$$66p_1 = 24 \qquad \Rightarrow \qquad p_1 = \frac{4}{11}$$

- You can verify that this also holds if we use the market clearing condition for x_1.
- So, any p_1, p_2 such that $p_1/p_2 = 4/11$ is a Walrasian price that sets aggregate excess demand to zero.

- The Walrasian equilibrium allocation that results from $p_1 = 4/11, p_2 = 1$ is:

$$x_1^1 = 29/2,\ x_2^1 = 58/11,\ x_1^2 = 13/2,\ x_2^2 = 52/11$$

- You can verify that this is a feasible allocation.
- Finally, we can verify that this is in the core.
- Since each agent is solving his utility maximization problem, his MRS is equal to the price ratio, and therefore equal to the other agent's MRS.
- Each agent is maximizing utility, and by definition, cannot end up at a lower utility than what he started with (at his endowment).



- We can plot the allocation $(x_1^1 = 29/2, x_2^1 = 58/11)$ and verify it is in the core.

- Chapter 5.3 deals with adding production into the economy, which we will skip.
- The main result from this chapter is that if production sets are convex, then a Walrasian equilibrium is guaranteed to exist, and the First and Second Welfare Thorems hold as well.
- Now, let's start on Chapter 5.4, which applies the general equilibrium framework to "goods" that may depend on random outcomes of the world.
- This is the basis of the general equilibrium approach to finance and asset pricing.

Chapter 5.4: Contingent Plans

- So far, we have considered a market economy with ordinary goods, e.g. food, clothing, etc.
- But there is no reason why we must restrict our concept of a "good" to those things.
- We can also extend our concept of "good" to incorporate time and uncertainty.
- For example: suppose x_1 denotes apples, and x_2 denotes oranges.
- We can classify goods by *when they are consumed*. So, if we have 2 time periods (today and tomorrow), we can define 4 goods:
- x_{11} = "apples today", x_{12} = "apples tomorrow", x_{21} = "oranges today", x_{22} = "oranges tomorrow".
- For example, a *futures contract* is a financial instrument that specifies that the holder must buy or sell a certain quantity of a good at a certain time in the future (e.g. buy 1000 tons of grain in 6 months).

Uncertainty

- Suppose there is a feature of the world that is random (for example, the weather).
- Suppose there are two possible outcomes, or *states of the world*: rainy (denoted $s = 1$), and sunny (denoted $s = 2$).
- Then, we can index each good with the *state in which it is consumed*.
- For example, if $x =$ "umbrellas", then $x_1 =$ "umbrellas when it is rainy", and $x_2 =$ "umbrellas when it is sunny".
- Other examples:
  - "1 unit of grain when the harvest is good" vs. "1 unit of grain when the harvest is bad"
  - "1 unit of income in a recession" vs. "1 unit of income in an expansion"
- These goods can have different prices and different effects on a consumer's utility.

Contingent Commodities

- To generalize, we can add both time and uncertainty to goods.
- These are called "contingent commodities" or "contingent claims", because they are contingent on (i.e. take effect if) a certain event occurs.
- At each time t, there is a market for contingent claims, called a *spot market*.
- Goods (contingent claims) may be re-traded in each time period.
- All the results from the general equilibrium model carry over to this setting.
- However, there are some difficulties in interpreting a "time-specific" commodity.
- For example, what if bankruptcy occurs, or a failure to produce what has been agreed to in the past?
- For now, we will ignore these issues.

Finance Economy

- A *finance economy* combines the GE model with risk averse utility functions.
- Suppose there is only one consumption good, call it money, income or wealth.
- There are S total states of the world (i.e. outcomes), and let s denote an individual state of the world.
- A *financial asset* is a security that entitles its holder to a specified payout for each possible state of the world.
- Suppose that asset j is specified by: $r^j = (r_1^j, \ldots, r_S^j)^T$
- Whoever holds 1 unit of asset j will receive r_s^j at $t = 1$, if the state of the world happens to be s.
- A *storage asset* (e.g. cash) would be $(1, \ldots, 1)^T$.
- A *riskless bond* with nominal yield $1 + r$ would be $(1 + r, \ldots, 1 + r)^T$.
- An asset is called *risk-free* if it gives the same payoff in every state.

Arrow securities

- The simplest asset is one that pays 1 unit in exactly one state of the world s, and zero in all other states.

$$e^s = (0, 0, ..., 1, ...0)^T$$

- This is called the *Arrow security* for state s.
- Any financial asset can be represented as a linear combination of Arrow securities.
- For example, suppose there are two states $s = 1, 2$ and two Arrow securities $e^1 = (1, 0)$ and $e^2 = (0, 1)$.
- Cash has a payoff of $(1, 1)$ and can be replicated by $e^1 + e^2$.
- A riskless bond with yield $1 + r$ has payoff $(1 + r, 1 + r)$ and can be replicated by $(1 + r)e^1 + (1 + r)e^2$.
- A risky asset with payoff (r_1, r_2) can be replicated by $r_1 e^1 + r_2 e^2$.

Two-Period Economy

- By convention, we will say that $t = 0$ is state $s = 0$, and the states at $t = 1$ are $s = 1, 2, ..., S$.
- Let w^s denote the amount of wealth in state s.
- Assume agents have a utility function

$$u(w_0) + \delta E\left[u(w_s)\right] = u(w_0) + \delta \sum_{s=1}^{S} \pi_s u(w_s)$$

- π_s is the probability that state s occurs.
- $\delta \in (0, 1)$ is the discount factor.
- This type of utility function is *time-separable*, i.e. additive in the utility for $t = 0$ and $t = 1$.

Efficient Risk-Sharing

- Suppose there are two agents ($i = 1, 2$) and two states ($s = 1, 2$), and agents are risk-averse: $u^i(\cdot)$ is strictly concave.
- Agents are endowed with some amount of securities that pay off at $t = 1$.
- Assume there is no *aggregate* risk: the sum of endowments for each state s is constant.
- There may be *idiosyncratic* risk: the endowment for an individual agent may differ across states.
- The *mutuality principle* states that an efficient allocation in this situation will diversify away idiosyncratic risk.
- Agents will consume the same amount in both states; they will only bear aggregate risk.

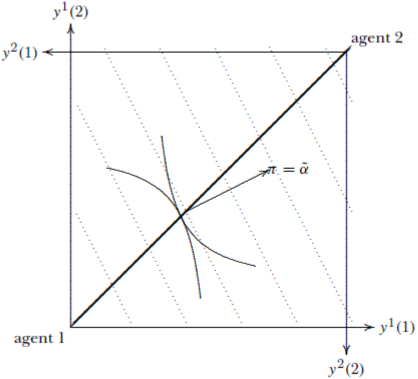


Figure 5.1. An Edgeworth box with no aggregate risk and full insurance. The dotted lines are iso-expected wealth lines, the fat line is the contract curve.

- The x and y axes are the total amount of income available in state $s = 1, s = 2$, respectively.
- Under the assumption of no aggregate risk (i.e. total income is the same in each state), the Edgeworth box is a square.
- The 45-degree line represents allocations where both agents have the same income in each state (perfect insurance).

Q7

- Suppose each consumer's utility over wealth is $u(w) = \ln(w)$.
- The probability of state $s = 1$ is p, and the probability of state $s = 2$ is $1 - p$.
- There is no consumption at $t = 0$, so each consumer's expected utility is
$$E[u(w)] = pu(w_1^i) + (1-p)u(w_2^i)$$
- There are two "goods": wealth in state $s = 1$, and wealth in state $s = 2$.
- We can view the expected utility function as a utility function over two goods w_1^i, w_2^i.
- Each good is an Arrow security, i.e. a financial asset that pays out 1 unit of wealth in one state, and zero in all other states.
- Suppose $(w_1^1, w_2^1) = (1, 3)$, $(w_1^2, w_2^2) = (3, 1)$

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Q7

- As before, we can define the contract curve.
- Each agent's MRS is:

$$MRS = \frac{\partial u / \partial w_1}{\partial u / \partial w_2} = \frac{p / w_1}{p / w_2} = \frac{p}{1-p} \frac{w_2}{w_1}$$

- Note that the MRS is similar to that of Cobb-Douglas utility $u(w_1, w_2) = w_1^p w_2^{1-p}$.
- The expected utility function is an increasing transformation of Cobb-Douglas utility, so the Marshallian demand function is the same.
- Then, you can proceed as in Exercise 5.11.

Q7

- You should find that both agents are perfectly insured (that is, consume the same amount in each state).
- The agents are insuring each other.
- This is only possible because the "good" state for agent 1 is the "bad" state for agent 2, and vice versa.
- This is an example of *mutual insurance*: a group of consumers pool their resources to insure each other, instead of going to an outside party like an insurance company.

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