

Topics in Bank Management: Lecture 3

Ronaldo Carpio

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Last Week

- ▶ We want to come up with something that banks can do, that individual investors cannot do for themselves.
- ▶ We saw two models where *coalitions* of depositors or borrowers can diversify away risks that an individual agent cannot.
- ▶ Ch 2.2: banks as *pools of liquidity* that diversify against liquidity shocks.
- ▶ Ch 2.3: holding companies as *coalitions of borrowers* that decrease the cost of borrowing, due to asymmetric information (the borrower knows the quality of his project, while the lender does not).
- ▶ This week, we'll look at another model in which asymmetric information introduces inefficiency, and how a bank that deals with many borrowers can decrease this inefficiency.

A Brief Review of the Principal-Agent Problem

- ▶ From Jehle & Reny, Chapter 8.2
- ▶ A *principal* is any entity that has a stake in an action taken by an *agent*, but the agent's action cannot be observed by the principal.
- ▶ Examples:
 - ▶ Owner of firm vs. employee: employee can choose to work hard or not
 - ▶ Owner of firm vs. manager: manager can choose a risky project or not
 - ▶ Insurance company vs. consumer: consumer can choose to drive safely or not
 - ▶ Lender vs. Borrower: borrower can choose to misreport cash flow or not
- ▶ Let's review the example in Ch. 8.2 of an insurance company vs. a consumer of insurance.

Example of Principal-Agent Problem

- ▶ There is a single insurance company and a single consumer who is considering whether to buy car insurance.
- ▶ The consumer might get into an accident, causing a random loss of ℓ , where $\ell \in \{0, 1, \dots, L\}$.
- ▶ The consumer can choose two levels of "effort" to drive safely: $e \in \{0, 1\}$. 1 gives a higher disutility to the consumer.
- ▶ The probability of getting into an accident of loss ℓ depends on e : $\pi_\ell(e)$, assumed to be strictly positive for all ℓ, e .
- ▶ Assume $\frac{\pi_\ell(0)}{\pi_\ell(1)}$ is strictly increasing in $\ell \in \{0, 1, \dots, L\}$.
- ▶ This means that conditional on accident ℓ being observed, the probability that $e = 0$ was chosen is higher than the probability that $e = 1$ was chosen.

Example of Principal-Agent Problem

- ▶ Assume the insurance company is risk-neutral. Therefore, it only cares about the expected value of its payoff.
- ▶ Assume the consumer has vN-M utility function $u(\cdot)$, which is strictly increasing and strictly concave.
- ▶ This implies the consumer is risk-averse, and is always willing to accept some decrease in average payoff in exchange for a decrease in variance.
- ▶ Let $d(e)$ denote the disutility of effort. Assume $d(1) > d(0)$.
- ▶ Consumer's vN-M utility function is: $u(\cdot) - d(e)$
- ▶ Assume the consumer's initial wealth is w , which is large enough to pay for insurance.

Insurance Contract

- ▶ A *contract* specifies an action in each observable state of the world (i.e. each random outcome that is observable to all parties).
- ▶ The insurance company can observe the loss ℓ , but not the effort e . Therefore, the contract can only specify actions for values of ℓ .
- ▶ The contract is a tuple $(p, B_0, B_1, \dots, B_L)$, where:
 - ▶ p is the price of the contract, paid by the consumer to the insurance company in the beginning
 - ▶ B_ℓ specifies the payment from the insurance company to the consumer, if accident ℓ occurs
- ▶ Assume that consumers have a *reservation utility* \bar{u} that they can get elsewhere (perhaps from a competing insurance company).
- ▶ We want to find a contract that maximizes the insurance company's profits, while still being acceptable to the consumer.

Symmetric Information Case (e is observable)

- ▶ Suppose e is observable. Then e can be specified in the contract.
- ▶ The insurance company wants to solve the following problem:

$$\max_{e, p, B_0, \dots, B_L} p - \sum_{\ell=0}^L \pi_{\ell}(e) B_{\ell} \quad \text{subject to}$$

$$\sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \geq \bar{u}$$

- ▶ This problem maximizes over the variables (e, p, B_0, \dots, B_L) . We can decompose it into two steps:
 - ▶ Take e as given, and maximize over (p, B_0, \dots, B_L) , to get a function of e .
 - ▶ Maximize this function over e .

Symmetric Information Case (e is observable)

- ▶ Take e as given.

$$\max_{p, B_0, \dots, B_L} p - \sum_{\ell=0}^L \pi_{\ell}(e) B_{\ell} \quad \text{subject to}$$

$$\sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \geq \bar{u}$$

- ▶ Lagrangian is:

$$L(p, B_0, \dots, B_L) = p - \sum_{\ell=0}^L \pi_{\ell}(e) B_{\ell} - \lambda \left[\bar{u} - \sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \right]$$

First Order Conditions

- First order conditions:

$$\frac{\partial L}{\partial p} = 1 - \lambda \left[\sum_{\ell=0}^L \pi_{\ell}(e) u'(w - p - \ell + B_{\ell}) \right] = 0$$

$$\frac{\partial L}{\partial B_{\ell}} = -\pi_{\ell}(e) + \lambda \pi_{\ell}(e) u'(w - p - \ell + B_{\ell}) = 0 \quad \text{for } \ell \in \{0, \dots, L\}$$

$$\frac{\partial L}{\partial \lambda} = \bar{u} - \sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) = 0$$

- The insurance company will lower the payouts as far as possible, so we can assume the constraint is binding and $\lambda > 0$.
- The middle equalities implies $u'(w - p - \ell + B_{\ell}) = 1/\lambda$ for all ℓ , therefore $B_{\ell} - \ell$ is *constant* for all ℓ .
- This is a classic risk-sharing result:
 - the risk-averse agent will fully insure (receive the same utility in all states/outcomes)
 - the risk-neutral party will take on risk (receive different utilities in different states/outcomes)

Optimal Contract for Symmetric Information

- ▶ There are many possible optimal contracts, since p can be adjusted to compensate for B_ℓ .
- ▶ One possible optimal contract is: $B_\ell = \ell$ for $\ell = 0, \dots, L$
- ▶ For each e , the optimal p is determined by the equation $u(w - p(e)) = d(e) + \bar{u}$
- ▶ Then, the optimal e maximizes the expected profit of the insurance company:

$$p(e) - \sum_{\ell=0}^L \pi_\ell(e) \ell$$

Asymmetric Information Case

- ▶ Suppose e is not observable. The insurance company must design the contract so that the consumer will voluntarily choose the company's desired e .
- ▶ This introduces an additional *incentive constraint*, that the choice of e is utility-maximizing for the consumer:

$$\max_{e, p, B_0, \dots, B_L} p - \sum_{\ell=0}^L \pi_{\ell}(e) B_{\ell} \quad \text{subject to}$$

$$\sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \geq \bar{u}$$

$$\sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \geq \sum_{\ell=0}^L \pi_{\ell}(e') u(w - p - \ell + B_{\ell}) - d(e')$$

- ▶ where $e \neq e'$.

Optimal Policy for $e = 0$

- ▶ In the symmetric information case, the optimal policy given $e = 0$ must satisfy the equality

$$u(w - p) = d(0) + \bar{u} \quad \text{for } \ell = 0, \dots, L$$

- ▶ Adding the incentive constraint to this problem does not increase the feasible set, so if a maximizer of the symmetric problem also satisfies the incentive constraint, then it is also a maximizer of the asymmetric case.
- ▶ The incentive constraint for $e = 0$ is:

$$\sum_{\ell=0}^L \pi_{\ell}(e) u(w - p - \ell + B_{\ell}) - d(e) \geq \sum_{\ell=0}^L \pi_{\ell}(e') u(w - p - \ell + B_{\ell}) - d(e')$$

$$d(0) \geq d(1)$$

- ▶ which is true by assumption.
- ▶ The optimal policy for $e = 0$ is the same as in the symmetric info case.

Optimal Policy for $e = 1$

- ▶ Form the Lagrangian with two constraints (the incentive constraint, and the constraint ensuring $e = 1$).
- ▶ It can be shown that both constraints are binding: consumer is indifferent between $e = 0$ and $e = 1$
- ▶ The optimal policy must have the property: $\ell - B_\ell$ is strictly increasing in ℓ
- ▶ This no longer provides full insurance. It specifies a deductible payment that increases with size of loss
- ▶ This is necessary to incentivize consumer to choose high effort.

2.4: Delegated Monitoring

- ▶ When there is asymmetric information, then efficiency can be increased with *monitoring*, which can mean:
 - ▶ *screening* of projects before deciding to lend to them
 - ▶ *preventing* opportunistic behavior of a borrower (e.g. replacing management who are not performing well)
 - ▶ *punishing* borrowers who fail to meet their contractual obligations
- ▶ Specialized firms are likely to be more efficient at monitoring than individual lenders. This provides a clear justification for the role of financial intermediaries.

Delegated Monitoring

- ▶ This is based on the paper: Diamond (1980), "Financial Intermediation and Delegated Monitoring"
- ▶ This paper has two parts: the first develops a principal-agent type of model, with the lender as the principal and the borrower/entrepreneur as the agent.
- ▶ The optimal contract between borrower and lender is derived, but it results in social inefficiency: there is a nonzero probability of wasteful punishment.
- ▶ This inefficient punishment can be diversified away by having an intermediary handle many borrowers, and form an optimal contract with lenders.
- ▶ For more on the use of optimal contracts in finance, you can look at the article on the course web site:
- ▶ Allen & Winton (1991), "Corporate Financial Structure, Incentives, and Optimal Contracting"
- ▶ We will examine more of these types of models later in the course.

Model Setup

- ▶ There are n identical, risk-neutral firms that are seeking to finance a project.
- ▶ Assume the riskless interest rate is R .
- ▶ Each firm requires an investment of 1, and the cash flow of each firm is an i.i.d. random variable \tilde{y} .
- ▶ All agents agree on the distribution of \tilde{y} , but it is only observed by the entrepreneur of the firm, but not the lender.
- ▶ Assume $\tilde{y} > 0$, and $E(\tilde{y}) > R + K$, where $K > 0$ will be defined later.
- ▶ The entrepreneur can lie and misreport the cash flow of the firm.

Approach 1: Optimal Financial Contract

- ▶ Similar to the principal-agent case, we want to find an optimal contract that both parties will voluntarily agree to.
- ▶ A *complete contract* is a rule that specifies what each party should do, for each observable state of the world.
- ▶ Here, only the entrepreneur's chosen repayment z is observable, so a *contract* specifies the actions of the borrower and lender for every possible z .

Approach 1: Optimal Financial Contract

- ▶ Suppose y is the actual cash flow of the firm (it is random, so this defines the state of the world).
- ▶ $z(y)$ is a function that specifies what the entrepreneur will repay, given y .
- ▶ $\phi(z)$ is the *penalty* that is imposed by the lender when the entrepreneur repays z .
- ▶ By assumption, penalty decreases the borrower's utility, but does not increase the utility of the lender. (e.g. debtors' prison, reputational damage)
- ▶ Therefore, it is welfare-reducing if the penalty is actually used.
- ▶ An optimal penalty function must satisfy these conditions:
 - ▶ Penalty must be high enough to deter underpayment
 - ▶ Penalty must be small to avoid welfare costs

Approach 1: Optimal Financial Contract

- ▶ The contract specifies:
 - ▶ $\phi(z)$, a (possibly infinite) set of numbers specifying the penalty imposed on borrower, conditional on observed repayment z
- ▶ The optimal contract maximizes the entrepreneur's expected return, given a minimum expected return to lenders of R .

$$\max_{\phi(z)} E \left[\max_z y - z - \phi(z) \right] \quad \text{subject to}$$

$$z \in \arg \max_z y - z - \phi(z)$$

$$E \left[\arg \max_z y - z - \phi(z) \right] \geq R$$

- ▶ Consider this penalty function: $\phi^*(z) = \max(h - z(y), 0)$, where h is a constant
- ▶ If $z < h$, then the penalty is $h - z$. Otherwise, the penalty is 0
- ▶ No incentive to lie about y (and underpay $z(y)$)

Approach 1: Optimal Financial Contract

- ▶ What should h be? Consider this value of h : the lowest value that provides lenders with an expected return of R .
- ▶ h is the smallest value that satisfies

$$P(\tilde{y} < h)E[\tilde{y}|\tilde{y} < h] + P(\tilde{y} \geq h)h = R$$

- ▶ Consider $z(y) = \min(h, y)$.
- ▶ The entrepreneur passes on all the cash flow if $\tilde{y} < h$; otherwise, he repays h and keeps the rest for himself
- ▶ This is like a standard credit contract: if the borrower cannot repay, the lenders seize the firm and liquidate the assets
- ▶ This contract is incentive-compatible for both the lender and the borrower:
 - ▶ the lender's expected return is equal to R , by the above equation
 - ▶ the borrower has no incentive to lie about \tilde{y} and underpay $z(\tilde{y})$

Approach 1: Optimal Financial Contract

- ▶ The final specification of the contract is:
 - ▶ The borrower agrees to repay h , which satisfies

$$P(\tilde{y} < h)E[\tilde{y}|\tilde{y} < h] + P(\tilde{y} \geq h)h = R$$

- ▶ If he repays some amount $z < h$, then the lender will impose a penalty of $h - z$, otherwise the penalty is zero
- ▶ This is not socially efficient, since the entrepreneur has to pay a penalty even when he is telling the truth.
- ▶ Entrepreneurs could be made better off without making lenders worse off, if \tilde{y} were *observable*.
- ▶ An alternative solution: Monitoring by lenders

Approach 2: Contract with Monitoring

- ▶ Suppose there are m lenders per firm, and that each lender can spend $K > 0$ to observe \tilde{y} .
- ▶ However, a lender who observes \tilde{y} only gains information for himself; other lenders will only be convinced if they also spend K
- ▶ Welfare is increased if $mK < E[\phi^*(z(y))]$, i.e. if the total cost of monitoring is less than the expected amount of penalty.
- ▶ Monitoring is preferable if m is small, and if the expected penalty is high (e.g. if there is a high probability of low returns)

Approach 3: Delegated Monitoring

- ▶ If monitoring is delegated to a financial intermediary (FI), monitoring costs can be reduced to K
- ▶ However, this brings up an additional problem: how can the lenders trust the FI? There must be an additional level of monitoring
- ▶ We can combine the two previous approaches: the lenders have an optimal contract with penalties between themselves and the FI
- ▶ And, the FI monitors the borrowers.
- ▶ If the FI deals with a large number of firms with independent returns, then the probability of low returns goes to 0, by the Law of Large Numbers
- ▶ Then, the expected penalties go to zero as well

Summary of Ch. 2

- ▶ We've seen 3 models where financial intermediaries can provide something that individual investors cannot do for themselves.
- ▶ In each of the 3 models, the individual borrower-lender relationship is inefficient, due to some sort of asymmetric information.
- ▶ If we assume that coalitions of borrowers or lenders can diversify away this inefficiency, then we have a rationale for financial intermediaries.

Ch. 3: Industrial Organization Approach to Banking

- ▶ Industrial Organization theory studies the organization of markets and industries.
- ▶ Issues of monopoly, oligopoly, competition, etc...
- ▶ Can model prices and quantities in equilibrium.
- ▶ However, this approach treats banks just like ordinary industrial firms
- ▶ The unique features of financial intermediaries (risk, bankruptcy, etc) are not captured

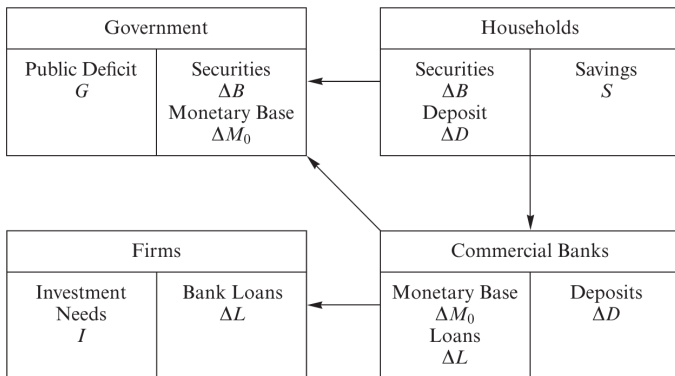
Perfectly Competitive Banks

- ▶ Banking activity is modeled like an industrial firm.
- ▶ Inputs are transformed into outputs via some production function, e.g. $Y = F(K, L)$
- ▶ In this chapter, inputs are deposits D , and loans L , which generate a cost of management $C(D, L)$
- ▶ Other authors following this approach sometimes model loans as the output, and deposits, labor as the input
- ▶ For examples, see Chapter 5 in *Handbook of Financial Intermediation and Banking*
- ▶ Basic conceptual difficulty: what are "inputs" and "outputs", how do you transform one into the other?

Perfectly Competitive Banks

- ▶ Inputs: Deposits D , Loans L
- ▶ Cost function: $C(D, L)$ is convex (decreasing returns to scale), twice differentiable
- ▶ N banks with same cost function
- ▶ Bank n 's balance sheet:
 - ▶ Assets: Reserves $R_n = C_n + M_n$
 - ▶ Liabilities: Deposits D_n
 - ▶ M_n : net position on interbank markets
 - ▶ C_n : cash reserves at the Central Bank
- ▶ α = coefficient of compulsory reserves: $C_n = \alpha D_n$

Perfectly Competitive Banks: Real Sector



- ▶ Δ refers to an increment in a stock value.
- ▶ B : Treasury bills
- ▶ M_0 : high-powered money (monetary base)
- ▶ money consists of deposits collected by banks: $D = \sum_n D_n$
- ▶ $M_0 = \sum_n C_n \alpha D$

Credit Multiplier Approach

- ▶ This is the usual description of monetary policy found in elementary macro textbooks and some banking textbooks.
- ▶ However, it no longer explains the US economy well, and current macroeconomic research focuses on the interest rate-setting role of the central bank.
- ▶ For more on this topic, see Woodford (2010), "Financial Intermediation and Macroeconomic Analysis"

Credit Multiplier Approach

- ▶ A change in monetary base M_0 or an open market operation (change in B) has a direct effect on money:

$$D = \frac{M_0}{\alpha} = \frac{G - B}{\alpha}$$

$$L = M_0 \left(\frac{1}{\alpha} - 1 \right) = (G - B) \left(\frac{1}{\alpha} - 1 \right)$$

- ▶ The *money multiplier* is defined by the effect of a marginal change in M_0 or B on the quantity of money in circulation:

$$\frac{\partial D}{\partial M_0} = -\frac{\partial D}{\partial B} = \frac{1}{\alpha} > 0$$

- ▶ Similarly, the credit multiplier is the effect on credit, L :

$$\frac{\partial L}{\partial M_0} = -\frac{\partial L}{\partial B} = \frac{1}{\alpha} - 1 > 0$$

Credit Multiplier Approach

- ▶ Issues with this approach:
- ▶ Banks are passive, make no decisions
- ▶ Monetary aggregates (e.g. M_0 , M_1 , M_2 ...) are no longer predictive, not used by most central banks
- ▶ Current monetary policy focuses on interest rate r

Competitive Behavior of Banks

- ▶ In perfect competition, firms are price takers: r_D, r_L, r are taken as given
- ▶ Profit maximization problem:

$$\max_{D,L} \Pi = r_L L + rM - r_D D - C(D, L)$$

- ▶ where the net interbank position $M = (1 - \alpha)D - L$

$$\Pi = r_L L + r(D - L - \alpha D) - r_D D - C(D, L)$$

- ▶ First order conditions:

$$\frac{\partial \Pi}{\partial L} = r_L - r - \frac{\partial C}{\partial L}(D, L) = 0$$

$$\frac{\partial \Pi}{\partial D} = r(1 - \alpha) - r_D - \frac{\partial C}{\partial D}(D, L) = 0$$

Competitive Behavior of Banks

- ▶ Each bank chooses D, L such that intermediation margins $r_L - r$ and $r(1 - \alpha) - r_D$ equal marginal management costs
- ▶ An increase in $r_D \rightarrow$ a decrease in bank's demand for deposits D
- ▶ An increase in $r_L \rightarrow$ an increase in bank's supply of loans L
- ▶ An increase in $r_L \rightarrow$ an increase in D iff $\frac{\partial^2 C}{\partial L \partial D} < 0$
- ▶ Interpretation of $\frac{\partial^2 C}{\partial L \partial D} < 0$: economies of scope.
- ▶ If $\frac{\partial^2 C}{\partial L \partial D} < 0$, an increase in L decreases marginal cost of deposits: more efficient to combine loans and deposits.

Competitive Equilibrium

- ▶ For each bank n :
 - ▶ Loan supply function: $L_n(r, r_L, r_D)$
 - ▶ Deposit demand function: $D_n(r, r_L, r_D)$
- ▶ Market clearing conditions:
 - ▶ Investment demand by firms: $I(r_L) = \sum_n L_n(r_L, r_D, r)$
 - ▶ Savings of households: $S(r_D) = B + \sum_n D_n(r_L, r_D, r)$
 - ▶ Interbank market: $\sum_n L_n(r_D, r_D, r) = (1 - \alpha) \sum_n D_n(r_L, r_D, r)$
- ▶ B is the net supply of Treasury bills, determined by central bank

Competitive Equilibrium

- Suppose $C(D, L) = \gamma_D D + \gamma_L L$, no economies of scope or scale. In equilibrium:
 - $r_D = r(1 - \alpha) - \gamma_D$
 - $r_L = r + \gamma_L$
 - r is determined by interbank condition:
 $I(r_L) = (1 - \alpha) [S(r(1 - \alpha) - \gamma_D) - B]$
- Macroeconomic effects of an increase in B : r increases

$$I'(r + \gamma_L) \frac{dr}{dB} = (1 - \alpha) \left[(1 - \alpha) S'(r(1 - \alpha) - \gamma_D) \frac{dr}{dB} - 1 \right]$$

$$\left[\underbrace{I'(r + \gamma_L)}_{<0} - (1 - \alpha^2) \underbrace{S'(r(1 - \alpha) - \gamma_D)}_{>0} \right] \frac{dr}{dB} = \underbrace{-(1 - \alpha)}_{<0}$$

Competitive Equilibrium

- ▶ An increase in B results in a decrease in loans and deposits. However, the absolute values are smaller than in the credit multiplier model:

$$\left| \frac{\partial D}{\partial B} \right| < 1, \left| \frac{\partial L}{\partial B} \right| < 1 - \alpha$$

- ▶ An increase in the reserve coefficient α results in an decrease in the volume of loans, but has an ambiguous effect on deposits

Monti-Klein Model of a Monopolistic Bank

- ▶ Monti (1972) "Deposit, credit, and interest rate determination under alternative bank objectives", Klein (1971) "A Theory of the Banking Firm"
- ▶ A monopolistic firm doesn't take prices as given, but faces the supply and demand curve.
 - ▶ Inverse loan demand function: $r_L(L)$
 - ▶ Inverse deposit supply function: $r_D(D)$
- ▶ Assume the interbank rate r is taken as given.
- ▶ Profit maximization:

$$\max D_L \Pi = [r_L(L) - r] L + [r(1 - \alpha) - r_D(D)] D - C(D_L)$$

- ▶ First-order conditions:

$$\frac{\partial \Pi}{\partial L} = r'_L(L)L + r_L(L) - r - \frac{\partial c}{\partial L}(D, L) = 0$$

$$\frac{\partial \Pi}{\partial D} = r(1 - \alpha) - r_D - r'_D(D)D \frac{\partial c}{\partial D}(D, L) = 0$$

Monti-Klein Model of a Monopolistic Bank

- ▶ The Lerner index $\hat{L} = \frac{P-MC}{P}$ describes a firm's market power. For a perfectly competitive firm, $\hat{L} = 0$.
- ▶ Lerner index is equal to the negative inverse of price elasticity of demand.
- ▶ Elasticities ϵ_L, ϵ_D :

$$\epsilon_L = -\frac{r_L L'(r_L)}{L(r_L)}, \epsilon_D = \frac{r_D D'(r_D)}{D(r_D)}$$

- ▶ Solution r_L^*, r_D^* is characterized by:

$$\frac{r_L^* - r - \frac{\partial C}{\partial L}}{r_L^*} = \frac{1}{\epsilon_L(r_L^*)}$$
$$\frac{r(1-\alpha) - r_D^* - \frac{\partial C}{\partial D}}{r_D^*} = \frac{1}{\epsilon_D(r_D^*)}$$

Monti-Klein Model of a Monopolistic Bank

- ▶ If price elasticities increase, intermediation margins decrease.
- ▶ If management costs are additive (i.e. $C(D, L) = f(D) + g(L)$), the bank's decision problem is separable: optimal r_D is independent of loan market, and optimal r_L is independent of deposit market.

Oligopolistic Banks

- ▶ Suppose there are N banks.
- ▶ Banks compete on quantities of output, i.e. Cournot model of oligopoly.
- ▶ Assume $C(D, L) = \gamma_D D + \gamma_L L$
- ▶ Inverse loan demand function: $r_L(\sum_n L_n)$
- ▶ Inverse deposit supply function: $r_D(\sum_n D_n)$

Cournot Equilibrium

- ▶ In Cournot-Nash equilibrium, each firm chooses output that maximizes profit, taking other firms' output as given.
- ▶ Equilibrium is a vector of output quantities $(D_n^*, L_n^*)_n, n = 1 \dots N$ such that for each n , (D_n^*, L_n^*) maximizes profit, given the prices $r_L(\sum_n L_n), r_D(\sum_n D_n)$ where (D_{-n}^*, L_{-n}^*) are taken as given.
- ▶ There is a unique symmetric equilibrium where $L_n = L^*/N$ and $D_n = D^*/N$ for all n .
- ▶ L^*, D^* are determined by the first-order conditions:

$$r_L(L^*) - r + r'_L(L^*) \frac{L^*}{N} - \gamma_L = 0$$

$$r(1 - \alpha) - r_D(D^*) - r'_D(D^*) \frac{D^*}{N} - \gamma_D = 0$$

Cournot Equilibrium

- ▶ In the symmetric equilibrium:

$$\frac{r_L(L^*) - r - \gamma_L}{r_L(L^*)} = \frac{1}{N\epsilon_L(r_L(L^*))}$$

$$\frac{r(1 - \alpha) - r_D(D^*) - \gamma_D}{r_D(D^*)} = \frac{1}{N\epsilon_D(r_D(D^*))}$$

- ▶ Same as the monopoly case, but elasticities are multiplied by N
- ▶ Assuming constant elasticities, effect of interbank rate r on loan, deposit rates:

$$\frac{\partial r_L^*}{\partial r} = \frac{1}{1 - \frac{1}{N\epsilon_L}}, \quad \frac{\partial r_D^*}{\partial r} = \frac{1 - \alpha}{1 + \frac{1}{N\epsilon_D}}$$

Cournot Equilibrium

$$\frac{\partial r_L^*}{\partial r} = \frac{1}{1 - \frac{1}{N\epsilon_L}}, \frac{\partial r_D^*}{\partial r} = \frac{1 - \alpha}{1 + \frac{1}{N\epsilon_D}}$$

- ▶ As competition increases (N goes up), r_L becomes less sensitive to changes in r .
- ▶ r_D becomes more sensitive to changes in r .
- ▶ Market power leads to lower deposit rates and higher loan rates.
- ▶ Note that this result does not depend on anything specific to banking or financial intermediation; the same result would hold true for an industrial firm.
- ▶ Next week, we'll look at the effect of competition on risk-taking, which is more bank-related.

Next Week

- ▶ For next week, please read sections 2 and 3 of the survey article by Allen & Winton, Ch. 3.5.1 and 3.6.1, and Ch. 4.1 - 4.2 in the textbook.
- ▶ I will post the first homework on the web site later today. It is due on 4/13 (no class on 4/6).