Topics in Bank Management: Lecture 4

Ronaldo Carpio

Mar. 30, 2015

Plan for This Lecture

- We will cover two more IO-related models today:
  - a model relating competition to risk-taking by banks;
  - a model of "relationship banking"
- Then, we will spend some time looking at the general equilibrium model of a financial market.
- As mentioned before, there are no intermediaries in this model, but it provides a basis for many of the intuitions in the banking models we've seen.

Ch 3.5.1: A Model of Risk-Taking on Investments

- What factors affect how much risk a bank takes on?
- Intuitively, we might expect that more competition leads to greater risk-taking by banks.
- Greater risk-taking may or may not be better for society as a whole.

Ch 3.5.1: A Model of Risk-Taking on Investments

- A bank has a quantity of loans, L, and deposits, D.
- We assume there is no equity; the balance sheet therefore requires $L = D$.
- r_D is the interest rate on deposits, chosen by the bank.
- r_L is the interest rate on loans, assumed to be a random variable.
- The *charter value* V of the bank is equal to the discounted value of expected future profits.
- In perfect competition, $V = 0$, since firms are assumed to make zero profit.
- If there are barriers to entry, V can be positive.

Ch 3.5.1: A Model of Risk-Taking on Investments

- Risk is captured by the random return on loans, r_L.
- Assume that the level of risk can be measured by a single parameter, σ, chosen by the bank.
- If σ increases, then the distribution of r_L becomes more risky in the sense of *second-order stochastic dominance* (Grossman & Stiglitz 1970):
  - the expected value remains constant: $E(r_L|\sigma) = \mu$ for all σ;
  - For any increasing convex function $h(\cdot)$, $\sigma' \geq \sigma \Rightarrow E(h(r_L)|\sigma') \geq E(h(r_L)|\sigma)$.
- This is called a *mean-preserving spread*.
- Equivalent to saying that the variance of r_L increases as σ increases.
- Any agent with a risk-averse (i.e. concave) utility function will prefer a lower σ to a higher one.

Case 1: Perfect Information

- First, assume that σ is observed by depositors, and r_D is set after σ is chosen by bank.
- If $r_L < r_D(\sigma)$, then the bank fails and cannot repay the depositors.
- Let $p(\sigma) = \text{Prob}(r_L > r_D(\sigma)|\sigma)$ denote the probability of success.
- If this happens, we assume the depositors seize the bank's assets and obtain return r_L by liquidating the assets.
- Depositors are assumed to be risk-neutral, and therefore will participate only if their net expected return is ≥ 1.

$$E\left[\min(r_L, r_D(\sigma)|\sigma)\right] \geq 1$$

Case 1: Perfect Information

- Let $\Pi(\sigma)$ denote the expectation of the bank's profits, given σ.

$$\Pi(\sigma) = D \cdot E\left[\max(0, r_L - r_D(\sigma)|\sigma)\right]$$

- The bank will choose σ to maximize $\Pi(\sigma) + p(\sigma)V$, under the participation constraint of depositors.
- Since lowering the returns to depositors increases profits, the constraint will be binding (i.e. hold with equality) at the optimum.

$$E\left[\min(r_L, r_D(\sigma)|\sigma)\right] = 1$$

Case 1: Perfect Information

$$\frac{\Pi(\sigma)}{D} = E\left[\max(0, r_L - r_D(\sigma)|\sigma)\right]$$

$$E\left[\min(r_L, r_D(\sigma)|\sigma)\right] = 1$$

- Combine these two equations to get:

$$\frac{\Pi(\sigma)}{D} + 1 = E\left[\min(r_L, r_D) + \max(0, r_L - r_D)|\sigma\right]$$

  - If $r_L > r_D$, then $\min(r_L, r_D) + \max(0, r_L - r_D) = r_D + r_L - r_D = r_L$.
  - If $r_L < r_D$, then $\min(r_L, r_D) + \max(0, r_L - r_D) = r_L + 0 = r_L$.
- We get:

$$\Pi(\sigma) + D = E(r_L)D = \mu D$$

Case 1: Perfect Information

$$\Pi(\sigma) + D = E(r_L)D = \mu D$$

- Note that $\Pi(\sigma) = (\mu - 1)D$ is independent of σ.
- The bank is risk-neutral, so the variance of r_L has no effect on its utility, except through the possibility of bank failure.
- If $V > 0$, the bank will minimize $p(\sigma)$ and choose the least risky level of σ.
- If $V = 0$, the level of risk σ is indeterminate.
- This model assumes that future profits V are independent of the risk level σ, which is not true in general.

Case 2: Imperfect Information

- Now, suppose that r_D is chosen by depositors without knowing σ.
- Depositors assume that banks will choose σ to maximize its profits.
- Let $\hat{\sigma}$ be the level of risk that depositors believe the bank will choose.
- Depositors will require a return $r_D(\hat{\sigma})$, that gives them at least a zero expected return.
- Let σ^* be the optimal level of risk for the bank, which solves the problem:

$$\max_{\sigma} \Pi(\sigma) + p(\sigma)V$$

- If we assume an equilibrium with rational expectations, then $\hat{\sigma} = \sigma^*$.

Case 2: Imperfect Information

- The bank's profit is

$$\Pi(\sigma) = D \cdot E\left[\max(0, r_L - r_D(\hat{\sigma}))|\sigma\right]$$

- If we take $\hat{\sigma}$ as fixed, then this is convex and increasing in σ.
- Therefore, in the competitive case, $V = 0$, and the banks take the maximum level of risk. Depositors anticipate this.
- If $V > 0$, there is some threshold $\hat{V}$ such that:
  - If $V < \hat{V}$, the bank chooses a maximum level of risk.
  - If $V > \hat{V}$, the bank chooses a minimum level of risk.

Summary of Model

- Increasing σ increases the variance of returns on loans, r_L.
- There is a tradeoff between short-term profits and long-term charter value V.
- Increasing σ increases short-term profits, but increases the probability of losing V.
- If σ is observable, $V > 0$ gives bank incentive to choose minimal level of risk.
- If σ is not observable, the bank will choose the maximum level of risk, unless V is large enough to deter it.

Implications

- Increasing transparency decreases managers' incentives to take risks, since the depositors will not participate.
- Imposing capital requirements implies that the bank has a charter value, which also reduces incentives to take risks.
- Relationship banking also increases charter value, and decreases incentives to take risks.

3.6.1: A Model of Relationship Banking

- Time has two periods.
- Lenders and borrowers are risk-neutral.
- Risk-free interest rate is 0.
- Firms have a project that requires an investment of 1, and with probability p, yields y. With probability $1 - p$, yields 0.
- Each period, the firm can invest in a new project.
- Banks provide funding if they have an expected return of at least ρ, where $\rho = 1$ in perfect competition, and $\rho > 1$ if banks have some market power.
- Before providing a loan, banks must pay a monitoring cost M once for each firm.
- After paying this cost, future loans can be made to the same firm without any additional cost.
- This provides an *ex post* monopoly power to the bank (since its marginal cost becomes lower than its competitors).

3.6.1: A Model of Relationship Banking

- The bank will invest $1 + M$ in a firm in the first period.
- If the firm succeeds (with probability p), the bank will invest an additional 1. Otherwise, no further investment is made.
- The total expected investment is $1 + M + p$.
- The return on this investment is $\rho(1 + M + p)$. Assume that all banks will get this return.
- The repayments in each period are R_1 and R_2, respectively. A firm will repay only if it is successful.

$$pR_1 + p^2 R_2 = \rho(1 + M + p)$$

3.6.1: A Model of Relationship Banking

$$pR_1 + p^2 R_2 = \rho(1 + M + p)$$

- Suppose there is competition between banks after the first period.
- If a firm switches to another bank, that bank has to invest $1 + M$, and must get an expected repayment equal to a return of ρ:

$$R_2 = \frac{\rho(1 + M)}{p}$$

- Therefore, this is also the optimal amount to charge, for the incumbent bank.
- Then, we can solve for R_1:

$$R_1 = \frac{\rho(1 + M(1 - p))}{p} < R_2 = \frac{\rho(1 + M)}{p}$$

3.6.1: A Model of Relationship Banking

$$R_1 = \frac{\rho(1 + M(1 - p))}{p} < R_2 = \frac{\rho(1 + M)}{p}$$

- The bank uses its *ex post* monopoly power in the second period, while competition drives down rates in the first period.
- In the competitive case ($\rho = 1$), there will be a profit in the second period and a loss in the first period, with a total of zero profit.
- Relationship banking leads to a holdup situation in the last period: the agent that can block the transaction can extract a higher rent.
- This also implies that banks will finance more risky projects, because they will ask for a lower payment in the first stage.
- The existence of an information monopoly is usually thought to be inefficient. How might this be overcome?
- Credit bureaus and public credit registers: organizations for sharing credit information among their members.

General Equilibrium Model of Finance

- Let's revisit the simple 2-agent exchange economy (Jehle & Reny Ch. 5.1).
- There are two goods.
- Two agents with smooth, quasiconcave utility functions: $u^1(x_1^1, x_2^1)$ and $u^2(x_1^2, x_2^2)$
- Each agent has an initial endowment of each good: $\boldsymbol{e^1} = (e_1^1, e_2^1)$ and $\boldsymbol{e^2} = (e_1^2, e_2^2)$
- The total endowment of each good in this economy is $(e_1^1 + e_1^2, e_2^1 + e_2^2)$
- We can plot all possible allocations of goods among the two agents in an Edgeworth box.

General Equilibrium Model of Finance

- A Walrasian equilibrium is a price vector $\boldsymbol{p} = (p_1, p_2)$ and an allocation $(x_1^1, x_2^1), (x_1^2, x_2^2)$ such that:
  - Each agent's allocation (x_1^i, x_2^i) maximizes his utility, given prices $\boldsymbol{p}$ and initial wealth $\boldsymbol{p} \cdot \boldsymbol{e}^i$
  - Markets clear: $\sum_i x_m^i = \sum_i e_m^i$ for each good $m = 1, 2$.
- At equilibrium, the first order conditions for each agent require that the ratio of marginal utilities equals the price ratio:

$$\frac{\frac{\partial u^1}{\partial x_1}}{\frac{\partial u^1}{\partial x_2}} = \frac{\frac{\partial u^2}{\partial x_1}}{\frac{\partial u^2}{\partial x_2}} = \frac{p_1}{p_2}$$

General Equilibrium Model of Finance

- We can extend the general equilibrium framework to handle time and risk by expanding our definition of what a *good* is.
- We can add the *time of availability* to the definition of a good.
- For example: instead of two goods "apple" and "orange", we can define the good "1 apple today" and "1 orange, one week from now".
- We can also define a good to be *contingent on a random event*.
- Examples:
  - "umbrella when it is raining" vs. "umbrella when it is not raining"
  - "1 unit of grain when the harvest is good" vs. "1 unit of grain when the harvest is bad"
  - "income when there is a recession" vs. "income when there is an expansion"
- These are called *contingent claims*.

General Contingent Claim Economy

- Let S denote the possible *states of the world*, and $s \in S$ is an individual state of the world.
- Time is finite: $t = 0, 1, ..., T$
- For each t, there is a *partition* ϵ_t of S (i.e. decompose S into a collection of non-intersecting subsets)
- This partition represents the *information* about the state s available at time t.
- At t, all agents know that an event $e \in \epsilon_t$ has occurred, but not necessarily which individual state $s \in e$ has occurred.

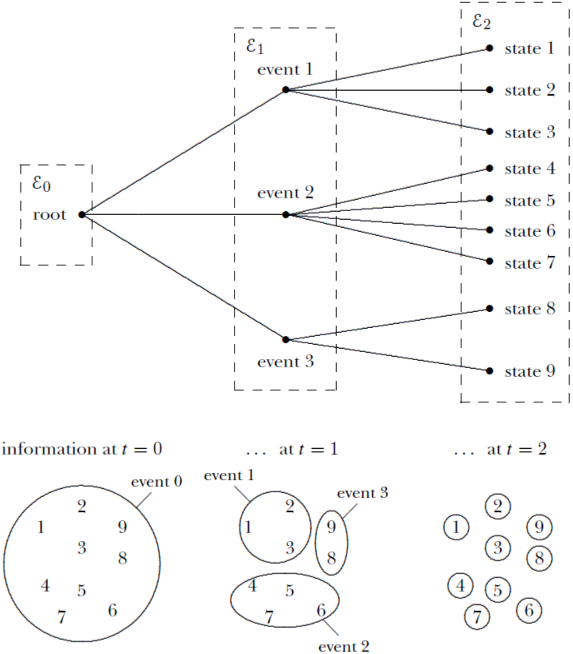

Figure 2.1. Resolution of uncertainty through time.

Contingent Claim Economy

- Claims may be contingent on a specific time and event.
- At each time t, there is a market for contingent claims, called a *spot market*.
- Goods (contingent claims) may be re-traded in each time period.
- All the results from the general equilibrium model carry over to this setting.
- However, there are some difficulties in interpreting a "time-specific" commodity.
- For example, what if bankruptcy occurs, or a failure to produce what has been agreed to in the past?
- For now, we will ignore these issues.

Two-Period Model

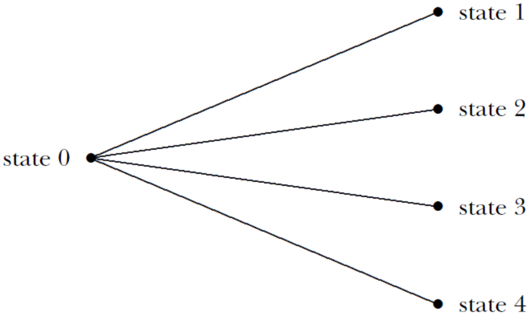

Figure 2.2. *A simple state tree.*

- At $t = 0$, there is complete uncertainty: the only information agents have is that all states of the world are possible.
- At $t = 1$, uncertainty is resolved: all agents know exactly which state of the world has occurred.

Finance Economy

- A *finance economy* combines the GE model with risk averse utility functions.
- Suppose there is only one consumption good, call it money, income or wealth.
- A *financial asset* is a security that entitles its holder to a specified payout for each possible state of the world.
- Suppose that asset j is specified by: $r^j = (r^j_1, \ldots, r^j_S)^T$
- Whoever holds 1 unit of asset j will receive r^j_s at $t = 1$, if the state of the world happens to be s.
- A *storage asset* (e.g. cash) would be $(1, \ldots, 1)^T$.
- A *riskless bond* with nominal yield $1 + r$ would be $(1 + r, \ldots, 1 + r)^T$.
- An asset is called *risk-free* if it gives the same payoff in every state.

Arrow securities

- The simplest asset is one that pays 1 unit in exactly one state of the world s, and zero in all other states.

$$e^s = (0, 0, ..., 1, ...0)^T$$

- This is called the *Arrow security* for state s.
- Any financial asset can be represented as a linear combination of Arrow securities.

Two-Period Economy

- By convention, we will say that $t = 0$ is state $s = 0$, and the states at $t = 1$ are $s = 1, 2, ..., S$.
- Let y^s denote the amount of consumption in state s.
- Assume agents have a utility function

$$v(y^0) + \delta E\left[v(y)\right] = v(y^0) + \delta \sum_{s=1}^{S} \pi_s v(y^s)$$

- pi_s is the probability that state s occurs.
- $v(\cdot)$ is a vNM utility function.
- $\delta \in (0, 1)$ is the discount factor.
- This type of utility function is *time-separable*, i.e. additive in the utility for $t = 0$ and $t = 1$.

Efficient Risk-Sharing

- Suppose there are two agents, $S = \{1, 2\}$, and agents are risk-averse: $v^i(\cdot)$ is strictly concave.
- Agents are endowed with some amount of securities that pay off at $t = 1$.
- Assume there is no *aggregate* risk: the sum of endowments for each state s is constant.
- There may be *idiosyncratic* risk: the endowment for an individual agent may differ across states.
- The *mutuality principle* states that an efficient allocation in this situation will diversify away idiosyncratic risk.
- Agents will consume the same amount in both states; they will only bear aggregate risk.

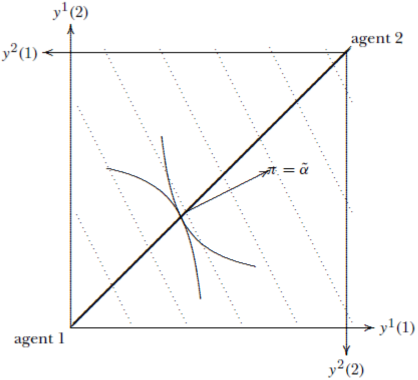

Figure 5.1. *An Edgeworth box with no aggregate risk and full insurance. The dotted lines are iso-expected wealth lines, the fat line is the contract curve.*