CUR 412: Game Theory and its Applications

Homework #4

- Due Date: May 31.
- Everyone must individually write up their own answers.
- Please write your names in English.

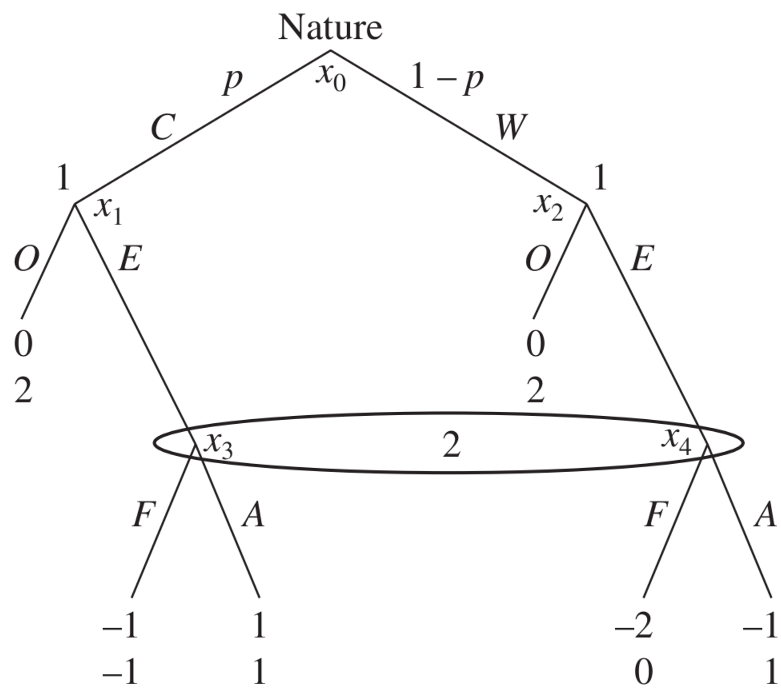
Q1: (Monty Hall problem) A contestant is on a TV game show, and is presented with three doors. Behind one of the doors is a car; behind the other two doors, a goat. The contestant values a car more than a goat. The sequence of actions is as follows:

1. The contestant picks a door, but the contents are not revealed.
2. The host, who knows where the car is located, opens one of the other two doors, revealing there is a goat inside.
3. The contestant now has the option of switching the door he has chosen. Should he switch?

We will state this problem using conditional probability. Let A be the event that the contestant's chosen door holds a car. Let B be the event that the host opens one of the doors, revealing a goat. Then, we want to find the conditional probability $P(A|B)$. If this probability is $\geq \frac{1}{2}$, then the contestant should switch. Assume $P(A)$, the unconditional probability of the contestant choosing the door with the car, is $\frac{1}{3}$.

- Use Bayes' Rule to formulate $P(A|B)$. For the next four questions, calculate $P(A|B)$ using the given assumptions.
- Suppose the host *always* reveals a goat, no matter which door the contestant chooses.
- Suppose the host always chooses from the remaining two doors with equal probability, and this is independent of the contestant's choice (so if the contestant picked the car; he reveals a goat; if the contestant picked a goat, he reveals the car half of the time).
- Suppose the host only reveals a goat and offers to switch if the contestant picks a goat.
- Suppose the host only reveals a goat and offers to switch if the contestant picks a car.

Q2: Consider the following version of the entry game:



The sequence of actions is as follows:

1. Nature chooses the challenger (Player 1)'s type, which can be *weak* (W) or *competitive* (C), with probability p and $1-p$ respectively. A *weak*-type challenger will cost less for the incumbent to fight. The incumbent (Player 2) does not know what Nature chooses.
2. The challenger knows his type, and chooses *Enter* (E) or *Out* (O).
3. The incumbent observes the challenger's choice, but not his type, and chooses *Fight* (F) or *Acquiesce* (A).

Assume both players only use pure strategies. There are 4 pure strategies for Player 1 and 2 pure strategies for Player 2.

(a) Suppose $p = 1$ (Nature always chooses C). Calculate the 4×2 matrix of payoffs of the strategic form.

(b) Suppose $p = 0$ (Nature always chooses W). Calculate the 4×2 matrix of payoffs of the strategic form.

(c) Now, suppose $p = \frac{1}{2}$. Calculate the 4×2 matrix of *expected* payoffs of the strategic form.

Q3: Suppose a worker is deciding whether to get an MBA degree or not. Assume that a worker's skill level is measured by a single number and that getting a degree has no effect on worker skill. There are 2 types of workers, "high"- and "low"-skilled workers. The game proceeds as follows:

1. Nature chooses Player 1's type, which can be H or L, and only Player 1 knows his type (Player 2 cannot observe it). The probability of being type H is given by $p > 0$.

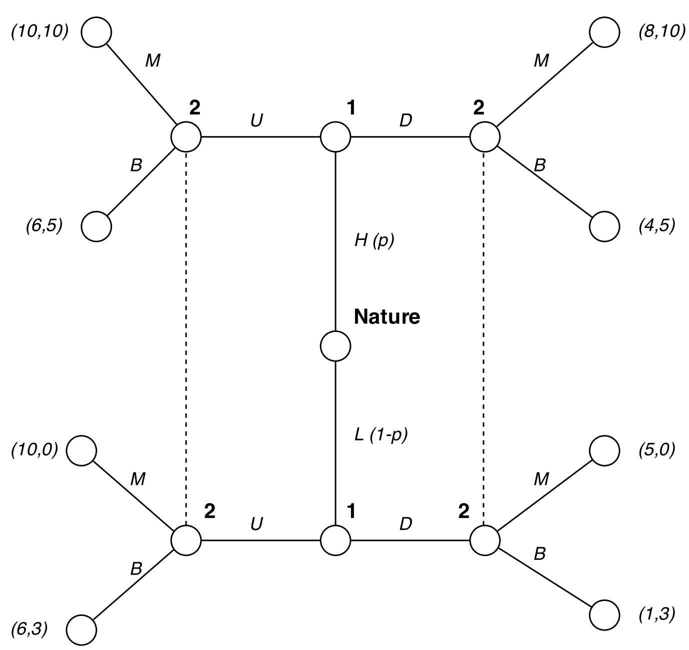
2. After Player 1 learns his type, he can choose D (get an MBA degree) or U (don't get a MBA degree). Getting an MBA degree requires a *type-dependent* cost. If a H-type player gets an MBA degree, he must pay a cost of c_H. A L-type player must pay a cost of c_L, with $c_H < c_L$. If the player does not get an MBA, the cost is 0.

3. Player 2 is an employer, who can assign Player 1 to one of two jobs: a manager (M) or a blue-collar worker (B), so Player 2's set of actions is $\{M, B\}$. The employer must pay a wage for each type of job; assume the wage for a manager is higher than the wage for a blue-collar worker, $w_M > w_B$.

4. Player 1's payoff is his wage minus the cost (if any) of getting a degree. Player 2's payoff (profits) is determined by the worker's skill level and the type of job, but not by the worker's choice of degree. The employer's profits are given by this table:

	M	B
H	10	5
L	0	3

Assume $p = \frac{1}{4}, c_H = 2, c_L = 5, w_M = 10, w_B = 6$. The complete game is given in this tree (note that the beginning of the game is in the center). We will find the set of weak sequential equilibria in pure strategies:



1. For each player, list their information sets and the histories in each information set.

2. Suppose Player 1's behavioral strategy profile is:

   - If my type is H, choose U with probability a and D with probability $1 - a$.

- If my type is L, choose U with probability b and D with probability $1-b$.

For each information set of Player 2, find the probability of reaching that information set, given this behavioral strategy.

3. For each information set of Player 2, find the beliefs over histories in that information set that would be consistent with this behavioral strategy.
4. Now, we will assume both players only use pure strategies. For each player, list their pure strategies (remember: a pure strategy specifies an action for every information set of the player).
5. List all possible outcomes (i.e. terminal histories) in which Player 1 chooses U.
6. Calculate the expected payoffs for all combinations of pure strategies (this should be a 4x4 matrix).
7. Find the Nash equilibria, if any.
8. Player 2 has 2 information sets, so his beliefs consist of 2 probability distributions: one over HU, LU and the other over HD, LD. For each strategy profile in part (7), find the set of beliefs that are consistent with the strategies (if any).
9. A *separating* equilibrium is when type H and type L workers choose different actions; a *pooling* equilibrium is when they choose the same action. Which of the NE in part (7) are separating, and which are pooling?