

CUR 412: Game Theory and its Applications, Lecture 1

Prof. Ronaldo CARPIO

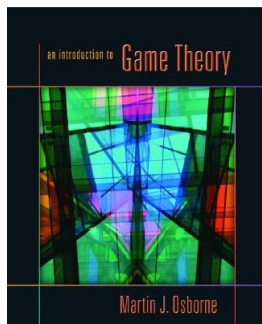
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Welcome to CUR412

- ▶ This course is an introduction to Game Theory, the study of *strategic* situations (i.e. situations with more than one decision-maker)
- ▶ This course will be taught entirely in English.
- ▶ Website: <http://rncarpio.com/teaching/CUR412>
- ▶ Announcements, slides, & homeworks will be posted on website

About Me: Ronaldo Carpio

- ▶ BS Electrical Engineering & CS, UC Berkeley
- ▶ Master's in Public Policy, UC Berkeley
- ▶ PhD Economics, UC Davis
- ▶ Joined School of Banking & Finance in 2012



- ▶ *An Introduction to Game Theory* (2003) by Martin Osborne
- ▶ A Chinese translation is available, it is recommended that you get it if you have difficulty following the textbook
- ▶ The first 3 chapters are available for download on the textbook webpage (see course website for details)
- ▶ Prerequisites: you should be familiar with optimization using derivatives, and basic probability

- ▶ Homework 15%, Midterm Exam 35%, Final Exam 50%
- ▶ Last semester's midterm and final exams are posted on the website
- ▶ Homework:
 - ▶ There will be 5 homework assignments, posted on the website
 - ▶ Write-ups must be individual, you may discuss the concepts in small groups
 - ▶ Exam problems will be similar to homework. The best way to prepare is to do the homework problems yourself
- ▶ Exam dates: to be announced

Contacting Me

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Course Outline

- ▶ Introduction & Motivation (Ch. 1)
- ▶ Static Games, Nash Equilibrium (Ch. 2)
- ▶ Nash Equilibrium: Applications (Ch. 3)
- ▶ Mixed Strategies & Mixed Strategy Equilibrium (Ch. 4)
- ▶ Extensive Form Games (Ch. 5)
- ▶ Sequential Games & Backwards Induction (Ch. 6, 7)
- ▶ Games with Imperfect Information (Ch. 10)
- ▶ Repeated Games (Ch. 14, 15)

What is Game Theory, and Why do we Need It?

- ▶ *Game Theory* is the mathematical study of *strategic* situations, i.e. where there is more than one decision-maker, and each decision-maker can affect the outcome.
- ▶ Previously in microeconomics, you studied *single-person* problems. For example:
 - ▶ How much of each good to consume, in order to maximize my utility?
 - ▶ How much output should a firm produce, in order to maximize profits?
- ▶ Rational behavior: choose the level that maximizes utility (or profits, or payoffs).
- ▶ However, in multi-agent situations, my choice may change your problem.
- ▶ We need a method that takes everyone's choices into account.

Examples of Strategic Situations

- ▶ Business
 - ▶ Competition between firms: price, quality, location...
 - ▶ Market segmentation by firm: offer different levels of quality
 - ▶ Auctions
- ▶ Political Science
 - ▶ Voting Strategically: always vote for your candidate, or vote to ensure your least preferred candidate loses?
- ▶ Sports
 - ▶ Tennis Serving
 - ▶ Soccer Penalty Kicks
- ▶ Biology
 - ▶ Why do animals confront each other, but rarely fight? (Hawk-Dove game)
 - ▶ Why does the peacock have a huge, costly tail? (Signaling, Handicap Principle)

Mathematical Definition of a Strategic (or Normal-Form) Game

- ▶ Terminology:
 - ▶ The decision-makers are called **players**.
 - ▶ Each player has a set of possible **actions**. A list of actions (e.g. the list of what players choose) is called an *action profile*.
 - ▶ Each player has *preferences* over the outcome of the game. The outcome is determined by the actions that all players have chosen. Some outcomes are more desirable than others.
- ▶ A *strategic game* is a model of interaction in which each player chooses an action *without knowing* what other players choose
- ▶ We can think of this as players choosing their actions *simultaneously*.

Mathematical Definition of a Strategic (or Normal-Form) Game

- ▶ We need to specify:
 - ▶ who the players are
 - ▶ what they can do
 - ▶ their preferences over the possible outcomes
- ▶ Definition: A *strategic game* consists of:
 - ▶ a set of *players*
 - ▶ for each player, a set of *actions*
 - ▶ for each player, *preferences* (i.e. a ranking) over all possible action profiles
- ▶ We will usually use *payoff functions* that represent preferences, instead of using preferences directly.

A 2-Player Static Game: The Prisoner's Dilemma

- ▶ Let's consider a specific example. Imagine this situation:
 - ▶ There are two suspects in a crime.
 - ▶ Each suspect can be convicted of a minor offense, but can only be convicted of a major offense if the other suspect finks (i.e. gives information to the police).
 - ▶ Each suspect can choose to be quiet or fink (inform).
 - ▶ If both stay quiet, each gets 1 year in prison.
 - ▶ If only one suspect finks, he goes free while the other suspect gets 4 years.
 - ▶ If both suspects fink, they both get 3 years.

Modeling the Prisoner's Dilemma

- ▶ Players: The two suspects.
- ▶ Actions: Each player's set of actions is Q, F .
- ▶ Preferences: We'll write down the action profile as: (Suspect 1's choice, Suspect 2's choice).
- ▶ Suspect 1's preferences, from best to worst:
 - ▶ $(F, Q) > (Q, Q) > (F, F) > (Q, F)$
- ▶ Suspect 2's preferences, from best to worst:
 - ▶ $(Q, F) > (Q, Q) > (F, F) > (F, Q)$
- ▶ Instead of using preferences directly, we will use a payoff function that assigns a utility to each outcome:
 - ▶ Suspect 1:
 $u_1(F, Q) = 3, u_1(Q, Q) = 2, u_1(F, F) = 1, u_1(Q, F) = 0$
 - ▶ Suspect 2:
 $u_2(F, Q) = 0, u_2(Q, Q) = 2, u_2(F, F) = 1, u_2(Q, F) = 3$

Bi-Matrix Form of Prisoner's Dilemma

		Player 2	
		Q	F
Player 1	Q	2,2	0,3
	F	3,0	1,1

- ▶ We can collect the payoff values into a *payoff matrix*:
- ▶ The two rows are the two possible actions of Player 1.
- ▶ The two columns are the two possible actions of Player 2.
- ▶ In each cell, the first number is the payoff of Player 1; the second is the payoff of Player 2.

Let's Play the Prisoner's Dilemma

- ▶ Everyone should have two cards: one Black and one Red card.
- ▶ How to play:
 - ▶ Start with two players, each with a Black and Red card.
 - ▶ Each player chooses to play Black or Red, and puts the card facedown.
 - ▶ Reveal both cards at the same time (why?)
- ▶ Suppose you are Player 1. If you play Red, then you get +2 and Player 2 gets +0.
- ▶ If you play Black, you get +0 and the other player gets +3.
- ▶ So, Red is beneficial to *you*, while Black benefits the *other* player.

	<i>Black</i>	<i>Red</i>
<i>Black</i>	3,3	0,5
<i>Red</i>	5,0	2,2

Modeling Other Situations as a Prisoner's Dilemma

- ▶ Suppose you are working with a friend on a joint project.
- ▶ Each of you can choose to *Work hard* or *Goof off* (be lazy).
- ▶ If the other person *Works hard*, each of you prefers to *Goof off*.
- ▶ Project would be better if both work hard, but not worth the extra effort.

		Player 2	
		<i>Work hard</i>	<i>Goof off</i>
Player 1	<i>Work hard</i>	2,2	0,3
	<i>Goof off</i>	3,0	1,1

Duopoly

- ▶ Two firms produce the same good.
- ▶ Each firm can charge a *High* price or a *Low* price.
- ▶ If both firms charge a high price, both get profit of 1000.
- ▶ If only one firm charges a high price, it loses customers, makes loss of 200. Other firm charges low price, gets profit of 1200
- ▶ If both firms charge low price, both get profit of 600.

	<i>High</i>	<i>Low</i>
<i>High</i>	1000, 1000	-200, 1200
<i>Low</i>	1200, -200	600, 600

Similarities to Prisoner's Dilemma

- ▶ Names of actions and payoffs are different, but *relative* payoffs are the same
- ▶ Preferences (i.e. ranking) over outcomes are the same as in Prisoner's Dilemma
- ▶ If both players cooperate, both get an outcome with good payoffs
- ▶ But if only one player chooses to defect, he gets an even better payoff (and cooperating player gets low payoff)

Applications of Prisoner's Dilemma

- ▶ Arms Race
 - ▶ Players: Countries
 - ▶ Actions: Arm, Disarm
- ▶ Provision of a Public Good
 - ▶ Players: Citizens
 - ▶ Actions: Contribute, Free-Ride
- ▶ Managing a Common Resource (Tragedy of the Commons)
 - ▶ Players: Animal Herders
 - ▶ Actions: Reduce Grazing, Overgraze

Bach or Stravinsky? (also known as Battle of the Sexes)

- ▶ Two people want to go to a concert by either *Bach* or *Stravinsky*.
- ▶ They prefer to go to the same concert, but one person prefers Bach while the other prefers Stravinsky.
- ▶ If they go to different concerts, each is equally unhappy.

	<i>Bach</i>	<i>Stravinsky</i>
<i>Bach</i>	2, 1	0, 0
<i>Stravinsky</i>	0, 0	1, 2

Let's Play Bach or Stravinsky

	<i>Black (Bach)</i>	<i>Red (Stravinsky)</i>
<i>Black (Bach)</i>	2, 1	0, 0
<i>Red (Stravinsky)</i>	0, 0	1, 2

Matching Pennies

- ▶ Prisoner's Dilemma and BoS have both conflict and cooperation. Matching Pennies is purely conflict.
- ▶ Each of two people chooses either *Head* or *Tail*.
- ▶ If the choices differ, Player 1 pays Player 2 \$1.
- ▶ If they are the same, Player 2 pays Player 1 \$1.
- ▶ Each person cares only about the money he receives.

	<i>Head</i>	<i>Tail</i>
<i>Head</i>	1,-1	-1,1
<i>Tail</i>	-1,1	1,-1

Let's Play Matching Pennies

	<i>Black (Head)</i>	<i>Red (Tail)</i>
<i>Black (Head)</i>	1,-1	-1,1
<i>Red (Tail)</i>	-1,1	1,-1

Solution Concept

- ▶ We've defined the game. What outcomes are more likely to occur?
- ▶ A *solution concept* (or *solution theory*) is a way of saying certain outcomes are less reasonable than others.
- ▶ A solution concept has two parts:
 - ▶ An assumption about the *behavior* of the players. We will assume rational behavior, i.e. choosing the action with the highest payoff
 - ▶ An assumption about the *beliefs* of the players.

- ▶ Suppose you are Player 1. In order to choose the best action, you need to have some idea of what Player 2 will choose
- ▶ This is called a *belief* about Player 2. Includes: the rules of the game, Player 2's payoff function, but also...
- ▶ Player 2 will also have a belief about you, which includes your beliefs about him, etc...
- ▶ Reasoning about what other players know (and what they know you know...) is called *higher-order knowledge*.
- ▶ We'll make a (very strong!) simplifying assumption: beliefs of all players are *correct*

- ▶ subscript i denotes player i or an action of player i
- ▶ subscript $-i$ denotes all other players except i , or their actions
- ▶ Action profile (i.e. a list of all actions chosen by all players)
 a^* is composed of a_i^* and a_{-i}^* :

$$a^* = (a_i^*, a_{-i}^*)$$

- ▶ a_i^* is the action chosen by player i
- ▶ a_{-i}^* is the set of actions chosen by everyone except player i

Nash Equilibrium

- ▶ This solution concept assumes that:
 - ▶ Players are rational (i.e. choose the highest payoff), given beliefs about other players
 - ▶ Beliefs of all players are correct
- ▶ We want to find an outcome that is a *steady state*, that is, starting from that outcome, no player wants to deviate.
- ▶ If an action profile a^* is a steady state, then all the players must *not* have other actions that they could play, that are *more preferable* to their current action in a^* .
- ▶ **Definition:** The action profile a^* in a strategic game is a **Nash Equilibrium** if, for every player i and every action b_i of player i , a^* is *at least* as preferable for player i as the action profile (b_i, a_{-i}^*) :

$$u_i(a^*) \geq u_i(b_i, a_{-i}^*) \quad \text{for every action } b_i \text{ of player } i$$

Nash Equilibrium

- ▶ Note that this definition does not guarantee that a game has a Nash equilibrium.
- ▶ Some games may have one, more than one, or zero Nash equilibria.

Prisoner's Dilemma

		Player 2	
		Q	F
Player 1	Q	2,2	0,3
	F	3,0	1,1

- ▶ (F, F) is the unique Nash equilibrium. No other action profile satisfies the conditions:
- ▶ (Q, Q) does not satisfy conditions, since $u_1(Q, Q) < u_1(F, Q)$
- ▶ $(F, Q) : u_2(F, Q) < u_2(F, F)$
- ▶ $(Q, F) : u_1(Q, F) < u_1(F, F)$
- ▶ Thus, the outcome predicted by the Nash equilibrium solution concept is that both players will *defect*.
 - ▶ Joint project: both will *Goof off*
 - ▶ Duopoly: both will charge a *Low* price (this is bad for the firms, but good for consumers)

Prisoner's Dilemma

	Q	F
Q	2,2	0,3
F	3,0	1,1

- ▶ Note that F is the best action for each player, *regardless* of what the other player does. (This is not the case in other games).
- ▶ However, (Q, Q) is a better outcome for both players than (F, F) .
- ▶ Individual rationality can lead to a socially inefficient outcome.
- ▶ How might players reach the better outcome, while still behaving rationally (payoff-maximizing)?
- ▶ Need to change the structure of the game, e.g.:
 - ▶ External: laws, contracts, reputation
 - ▶ Internal: emotions, social norms

	<i>Bach</i>	<i>Stravinsky</i>
<i>Bach</i>	2, 1	0, 0
<i>Stravinsky</i>	0, 0	1, 2

- ▶ Starting from $(Bach, Bach)$, no player can get a higher payoff by changing his action.
- ▶ Same for $(Stravinsky, Stravinsky)$.
- ▶ For $(Bach, Stravinsky)$ or $(Stravinsky, Bach)$, at least one player has an incentive to deviate.
- ▶ Two Nash equilibria: $(Bach, Bach)$ and $(Stravinsky, Stravinsky)$.
- ▶ Both outcomes are compatible with a steady state.

Matching Pennies

	<i>Head</i>	<i>Tail</i>
<i>Head</i>	1,-1	-1,1
<i>Tail</i>	-1,1	1,-1

- ▶ There is *no* Nash equilibrium.
- ▶ For every action profile, at least one player has an incentive to deviate.
- ▶ There will never be a steady state in this situation.

Next Week

- ▶ Next week, we will look at ways of finding Nash equilibria (if they exist).
- ▶ For next week's lecture, please read Chapter 1 and Chapters 2.1-2.5 in Osborne.
- ▶ If you don't have the textbook, check the course website for information on how to get the textbook chapters.