CUR 412: Game Theory and its Applications

Final Exam

Ronaldo Carpio

Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 120 minutes.
- There are 5 questions, for a total of 100 points.

Q1. **(16 pts.)** Answer whether each of the following statements is True or False. You don't have to provide an explanation.

(a) Every subgame perfect Nash equilibrium is a Nash equilibrium.

(b) In the Stackelberg duopoly game, the follower becomes better off than in the Cournot game, since she can move after observing the leader's strategy.

(c) If a static game is played repeatedly, then some outcome other than static Nash equilibria can possibly be achieved as a subgame perfect Nash equilibrium.

(d) A finite extensive game with perfect information may fail to have a subgame perfect equilibrium.

Q2. **(20 pts.)** Consider the following game played between a boss (B) and an employee (E). The boss offers a wage, $w \geq 0$. After observing the wage offer, the employee decides how much effort, $e \geq 0$, to expend. The payoff functions for the boss and the employee are (α is a constant, $\alpha \geq 0$):

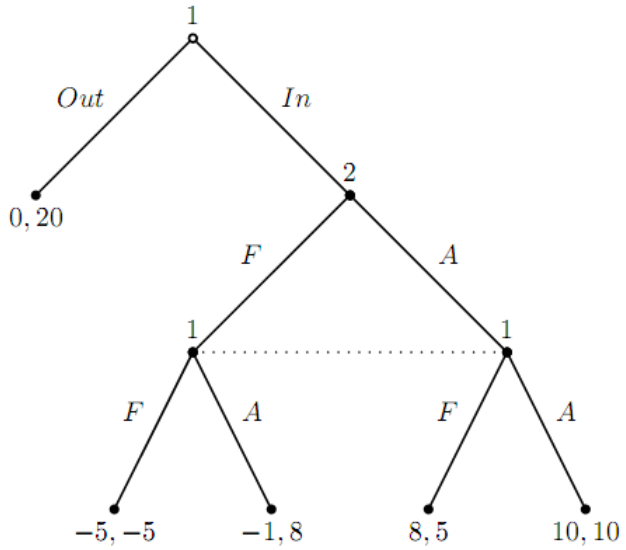
$$u_B(w, e) = 2\sqrt{e} - w$$

$$u_E(w, e) = w - \frac{e^2}{2} + \alpha we$$

(a) **(10 pts.)** What is the optimal effort for the employee, as a function of w?

(b) **(10 pts.)** What are the subgame perfect equilibrium choices of w and e, as a function of α?

Q3. **(20 pts.)** Find the set of pure strategy Nash equilibria and subgame perfect equilibria of the following game:



Q4. **(24 pts.)** Consider the infinitely repeated version of the following game:

	C	D
C	4,4	0,6
D	6,0	1,1

The payoff of player i to any infinite sequence of payoffs $\{u_{it}\}$ is given by the normalized discounted sum of payoffs:

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_{it}$$

where $0 < \delta < 1$.

(a) **(12 pts.)** For what values of δ, if any, does it constitute a subgame perfect equilibrium when both players choose this strategy?

- Choose C in period 1.
- Choose C after any history in which the previous period's outcome was (C, C).
- Choose D after any other history.

(b) **(12 pts.)** For what values of δ, if any, does it constitute a subgame perfect equilibrium when both players choose this strategy?

- Choose C in period 1.
- Do whatever your opponent did in the previous period.

Q5. **(20 pts.)** Consider the following signaling game. Nature (N) chooses the type of player 1 to be Tough (T) with probability 0.8, or Weak (W) with probability 0.2. Player 1 observes his type and chooses l or r. Player 2 observes only the action choice of player 1 but not the type, and chooses u or d. All these and the payoffs are common knowledge. Find the set of perfect Bayesian equilibria of this game.