CUR 412: Game Theory and its Applications

Midterm Exam

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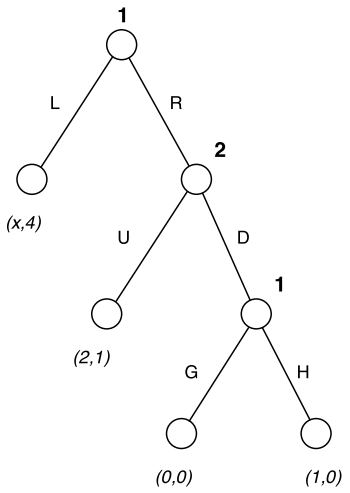
Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 120 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(24 pts)** Consider the following extensive form game:



(a) **(8 pts)** Suppose $x = 1$. Find the set of pure strategy Nash equilibria and subgame perfect Nash equilibria.

(b) **(8 pts)** Find the range of x for which (R, U) is the unique subgame perfect NE outcome.

(c) **(8 pts)** Find the range of x for which L is a Nash equilibrium outcome.

Q2. **(25 pts)** Suppose two people are working on a project and must decide how to split its value. Player 1 can exert an effort $e \geq 0$ with a cost $c(e) = 0.5e^2$. If Player 1 and Player 2 can agree on how to divide the project, then a total value of $v(e) = e$ is produced. If they cannot come to an agreement, the project produces less value, $y(e) = ke, 0 \leq k \leq 1$, that goes to Player 1 only. The game has three stages:

1. Player 1 chooses effort $e \geq 0$.
2. Player 2 observes e, chooses an offer $\alpha, 0 \leq \alpha \leq 1$.
3. Player 1 observes α, chooses to *Accept* or *Reject*.

If Player 1 *Accept*s, his payoff is $\alpha e - 0.5e^2$ and Player 2's payoff is $(1 - \alpha)e$. If Player 1 *Reject*s, his payoff is $ke - 0.5e^2$ and Player 2's payoff is zero.

(a) **(5 pts)** Draw the tree representation of this game.

(b) **(10 pts)** Find Player 1's choice of e in a subgame perfect NE.

(c) **(10 pts)** Suppose Player 2 could choose $k, 0 \leq k \leq 1$ *before* the start of the game. What would he choose? What would be Player 1's choice of e in SPNE?

Q3. **(24 pts.)** Consider the infinitely repeated version of the following game:

	H	D
H	1,1	3,0
D	0,3	2,2

The payoff of player i to any infinite sequence of payoffs $\{u_{it}\}$ is given by the normalized discounted sum of payoffs:

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}u_{it}$$

where $0 < \delta < 1$.

(a) **(12 pts.)** For what values of δ, if any, does it constitute a subgame perfect equilibrium when both players choose this strategy?

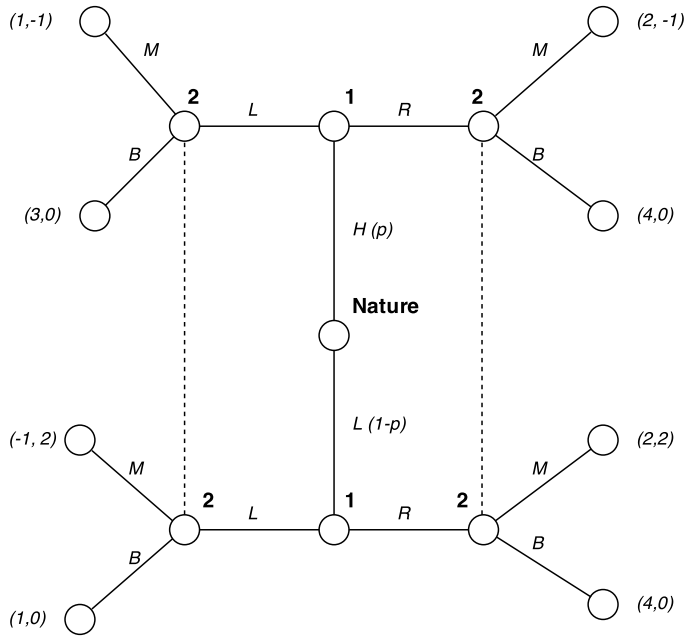
- Choose D in period 1.
- Choose D after any history in which both players have always played D.
- Choose H after any other history.

(b) **(12 pts.)** Suppose the game is modified to have the following payoffs:

	H	D
H	0,0	3,1
D	1,3	2,2

For what values of δ, if any, does it constitute a subgame perfect equilibrium when both players choose the strategy in part (a)?

Q4. **(27 pts.)** Consider this signaling game. Nature chooses H, L with probability $p = \frac{1}{2}$.



(a) **(3 pts)** For Player 1 and Player 2, list the histories in each player's information sets.

(b) **(4 pts)** For each of Player 1 and Player 2's information sets, list their pure strategies.

(c) **(10 pts)** Calculate the expected payoffs for all combinations of pure strategies (it should be a 4×4 matrix).

(d) **(10 pts)** Find the set of pure strategy weak sequential equilibria.