CUR 412: Game Theory and its Applications

Midterm Exam

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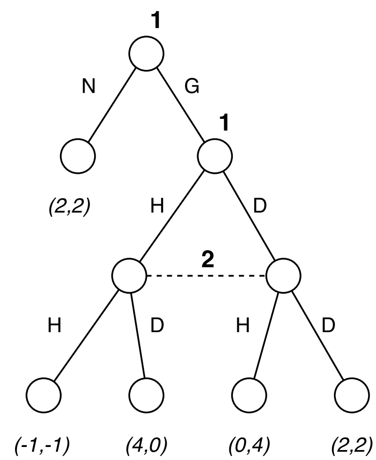
Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 120 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(24 pts)** Consider the following extensive form game:



(a) **(8 pts)** Write down the matrix for the strategic form of this game.

(b) **(8 pts)** Find all pure strategy Nash equilibria.

(c) **(8 pts)** Find all pure strategy subgame perfect NE.

Q2. **(24 pts)** Consider two firms that play a Cournot duopoly game with inverse demand $p = 100 - q$ and costs for each firm given by $c_i(q_i) = 10q_i$. Suppose that before the Cournot duopoly game, Firm 1 can choose to invest in *cost reduction.* If Firm 1 does, then it must pay a one-time cost of F, and its cost function drops to $c_1(q_1) = 5q_1$. If Firm 1 does not invest in cost reduction, there is no change.

(a) **(8 pts)** Write down the tree representation of this game.

(b) **(12 pts)** Find the value of F for which the unique subgame perfect NE has Firm 1 investing. Call this F^*.

(c) **(4 pts)** Assume that $F > F^*$. Find a Nash equilibrium of this game that is not subgame perfect.

Q3. **(24 pts.)** Consider the following Bertrand duopoly model. The demand function for each firm $i = 1, 2$ as a function of prices P_1, P_2 is:

$$Q_i(P_1, P_2) = \begin{cases} 2 - P_i & \text{if } P_i < P_j \\ \frac{2-P_i}{2} & \text{if } P_i = P_j \\ 0 & \text{if } P_i > P_j \end{cases}$$

Suppose costs are zero, so each firm's profit is

$$\pi_i(P_1, P_2) = P_i Q_i(P_1, P_2)$$

Let $P^m = 1$ be the price that would be chosen in monopoly (i.e. it maximizes $P(2 - P)$). Suppose that this game is repeated infinitely, in period $t = 1, 2, ...$ The payoff of firm i to the infinite sequence of profits $\{\pi_{i,t}\}$ is the discounted average (where $0 \leq \delta \leq 1$):

$$(1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \pi_{i,t}$$

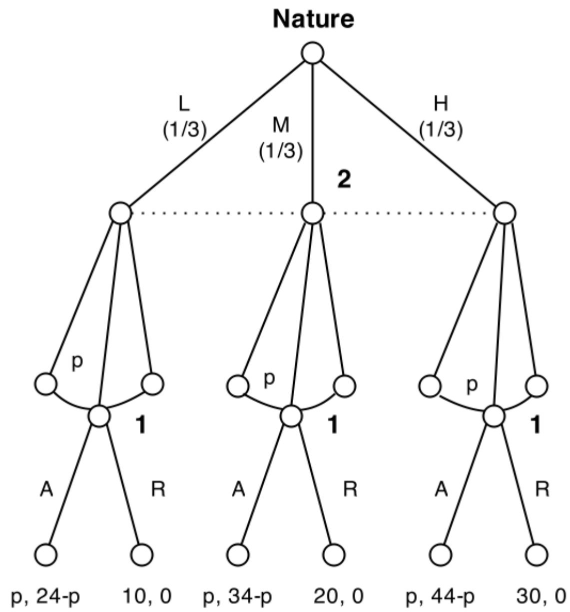
Consider this strategy profile:

- Choose P^m in the first period, and after any history in which both firms have always played P^m.
- Choose $P_i = 0$ after any other history.

(a) **(8 pts)** Calculate the 2x2 matrix of payoffs for the single stage game, where each firm chooses either P^m or 0.

(b) **(16 pts)** For what range of δ, if any, is the above strategy profile a SPNE in the infinitely repeated game?

Q4. **(28 pts.)** Consider this extensive-form game: Suppose Player 1 owns a car. Nature



chooses the quality of the car, which may be high (H), medium (M), or low (L), with equal probabilities of $\frac{1}{3}$ each. Player 1 knows what the result of Nature's choice is, but Player 2 does not. Player 2 is deciding how much to offer for the car. The value of the car to each player is:

$$v_1(q) = \begin{cases} 20 & \text{if } q = L \\ 30 & \text{if } q = M \\ 40 & \text{if } q = H \end{cases}$$

$$v_2(q) = \begin{cases} 24 & \text{if } q = L \\ 34 & \text{if } q = M \\ 44 & \text{if } q = H \end{cases}$$

The sequence of actions is:

1. Player 2 offers a price p to Player 1.
2. Player 1 chooses whether to *Accept* or *Reject*.
   - If Player 1 *Accepts*, Player 1's payoff is the price p, and Player 2's payoff is his valuation of the car minus the price p.
   - If Player 1 *Rejects*, Player 1's payoff is his valuation of the car, and Player 2's payoff is 0.

(a) **(8 pts)** Suppose Player 2 offers $p \geq 0$. Find the best response of Player 1, for each of the three possible quality levels.

(b) **(10 pts)** Show that there is no pure strategy weak sequential equilibrium in which the car is traded at a price equal to 30, the expected value of v_1.

(c) **(10 pts)** Find the range of p at which a trade can occur in a pure strategy weak sequential equilibrium.