

**CUR 412: Game Theory and its Applications**

**Midterm Exam**

Ronaldo Carpio

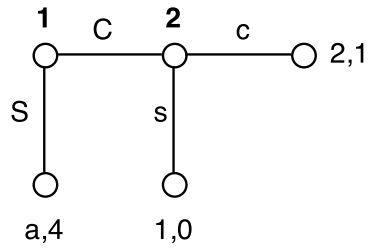
June 28, 2015

---

Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 120 minutes.
- There are 4 questions, for a total of 100 points.

Q1. (20 pts) Consider the following extensive form game:



- (a) (10 pts) Assume  $a = 1$ . Find the set of pure strategy NE and subgame perfect NE.
- (b) (5 pts) Find the range of  $a$  for which  $S$  is the unique subgame perfect equilibrium outcome.
- (c) (5 pts) Find the range of  $a$  for which  $(C, c)$  is the unique Nash equilibrium outcome.

Q2. (20 pts) Consider the following Cournot duopoly game. Firms 1 and 2 choose output levels  $q_1, q_2$ ; the profit function of firm  $i$  is:

$$\pi_i(q_1, q_2) = \begin{cases} q_i(1 - q_1 - q_2) & \text{if } q_1 + q_2 \leq 1 \\ 0 & \text{if } q_1 + q_2 > 1 \end{cases}$$

Firm 2 is run by its owner, while Firm 1 is run by a *manager* whose utility function is given by:

$$w(q_1, q_2) = \pi_1(q_1, q_2) + \alpha q_1$$

where  $0 \leq \alpha \leq 1$ . The sequence of actions is as follows:

1. First, the owner of firm 1 chooses  $\alpha \in [0, 1]$ , which is known by all players.
2. Second, the manager of firm 1 and the owner of firm 2 simultaneously choose  $q_1, q_2$ , respectively.

The owners of each firm want to maximize their profits,  $\pi_i$ . The manager wants to maximize his payoff  $w$ . Find the subgame perfect equilibrium levels of  $\alpha, q_1, q_2$ .

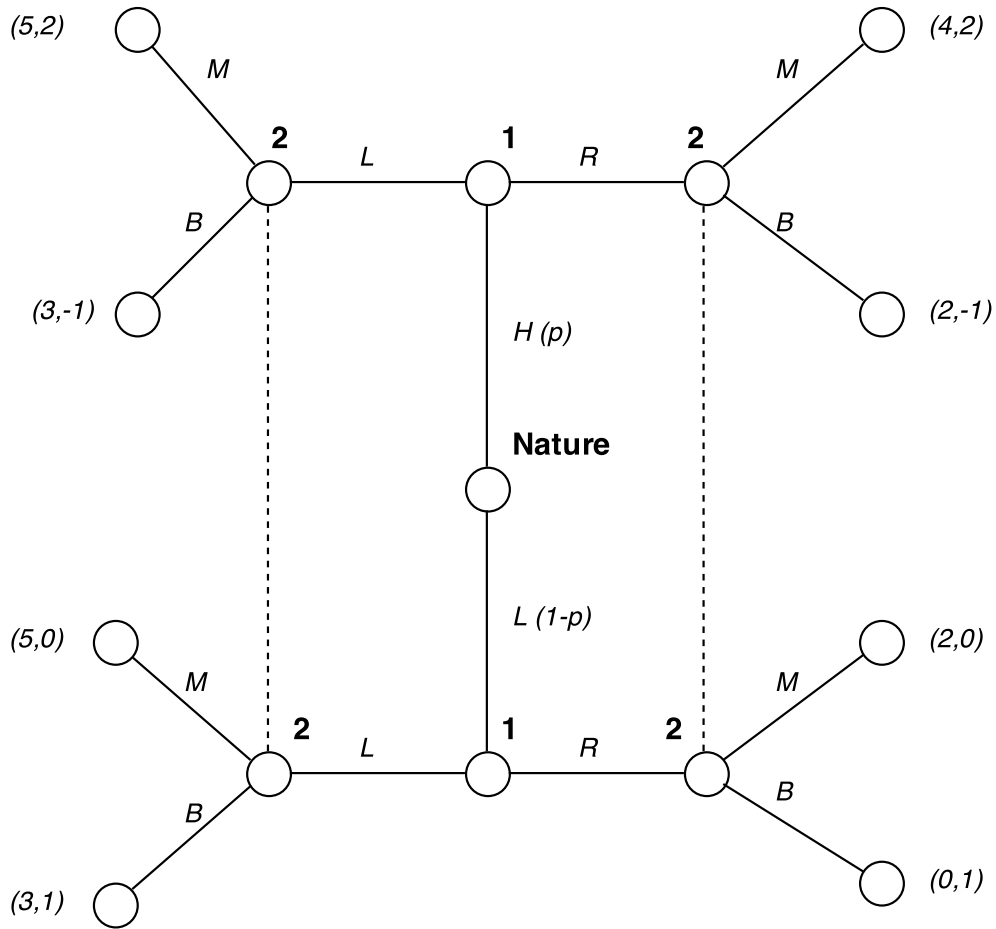
Q3. (30 pts.) Suppose two firms in a Cournot duopoly have zero unit cost and fixed cost. Each firm chooses  $q_1, q_2$ , respectively. Market demand is given by  $P = 200 - Q$ , where  $Q = q_1 + q_2$ .

- (a) (10 pts.) Find the Nash equilibrium levels of  $q_1, q_2$ , and firms' profits.
- (b) (5 pts.) Suppose both firms combined into a single monopolist. Find the equilibrium price and quantity.

Now, suppose this game is infinitely repeated, with discount factor  $\delta < 1$ . In each period, a firm can choose to:

- *Collude*, in which case the firm chooses to produce half of the monopolist's quantity in (b), or
  - *Defect*, in which case the firm maximizes its own profits, given the other firm's quantity.
- (c) (5 pts.) Write down the  $2 \times 2$  matrix of payoffs for a single stage of the repeated game.
- (d) (10 pts.) Find the range of  $\delta$  for which it is a subgame perfect Nash equilibrium when both firms play a modified grim trigger strategy:
- If *Defect* has never been played by either firm, then choose *Collude*.
  - If *Defect* has been played at any time in the past by either firm, then choose *Defect*.

Q4. (30 pts.) Consider this signaling game. Nature chooses  $H, L$  with probability  $p = \frac{1}{2}$ . Player 1's payoff is listed first in the pair of numbers for each outcome.



- (3 pts) For Player 1 and Player 2, list the histories in each player's information sets.
- (3 pts) For each of Player 1 and Player 2's information sets, list their pure strategies.
- (12 pts) Calculate the expected payoffs for all combinations of pure strategies (it should be a  $4 \times 4$  matrix).
- (12 pts) Find the set of pure strategy weak sequential equilibria.