CUR 412: Game Theory and its Applications

Midterm Exam

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Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 90 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(10 pts)**. Answer whether each of the following statements is True or False. You don't have to provide an explanation.

(a) If a game has a finite number of players and actions, there always exists a pure strategy Nash equilibrium.

(b) If two different actions are played (with positive probability) in a mixed strategy Nash equilibrium, then these actions give the same expected payoffs (given that the other players do not deviate from the equilibrium).

(c) In a 2-player strategic game with ordinal preferences, for any action a_i of player i, there exists at least one action for player j that is a best response to a_i.

(d) If an action is not strictly dominated in a strategic game without randomization, it cannot be strictly dominated when randomization is allowed.

Q2. **(30 pts)** Consider the following game:

	L	R
T	4,4	0,2
M	2,0	2,2
B	3,0	1,0

(a) **(15 pts)** Find the set of pure strategy Nash equilibria.

(b) **(15 pts)** Find a mixed strategy Nash equilibrium in which Player 1 plays all three actions with positive probability.

Q3. **(30 pts)** Two individuals, player 1 and player 2, choose how much effort to put into a joint project. If player 1 puts in effort level $x \geq 0$ and player 2 puts in effort $y \geq 0$, their payoffs are given by:

$$u_1(x, y) = (x + y + xy) - 2x^2$$

$$u_2(x, y) = v(x + y + xy) - 2y^2$$

where v is a constant, $0 \leq v \leq 2$.

(a) **(20 pts)** Find the set of Nash equilibria of this game as a function of v.

(b) **(10 pts)** Find an action profile that maximizes the sum of their payoffs.

Q4. **(30 pts)** Two people are trying to share a cake of size 100. They each simultaneously announce a real number from 0 to 100. If the sum of the numbers is less than or equal to 100, each person receives the number he announced. If the sum is greater than 100, then the player who announced the smaller number, say x, receives x while the other person receives $100 - x$. If the sum is greater than 100 and both numbers are the same, then each receives 50.

(a) **(10 pts)** Formulate this situation as a strategic game.

(b) **(10 pts)** Show that $(50, 50)$ is a Nash equilibrium.

(c) **(10 pts)** Is there any other Nash equilibrium? Prove your answer.