CUR 412: Game Theory and its Applications

Midterm Exam

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Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 90 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(10 pts)**. Answer whether each of the following statements is True or False. You don't have to provide an explanation.

For the following questions, assume we have a 2-player game with finite actions, and we have written down the payoffs in matrix form.

(a) If we add 10 to Player 1's payoff in every cell of the matrix, the set of pure Nash equilibria is unchanged.

(b) If we add 10 to Player 1's payoff in every cell of the matrix, the set of mixed Nash equilibria is unchanged.

(c) Suppose one of Player 1's actions is weakly dominated. If we delete this row from the matrix, the matrix that remains has the same set of pure Nash equilibria as the original matrix.

(d) Suppose one of Player 1's actions is strictly dominated. If we delete this row from the matrix, the matrix that remains has the same set of pure Nash equilibria as the original matrix.

(e) For any action a_1 of Player 1, there exists at least one action for Player 2 that is a best response to a_1.

Q2. **(30 pts)** Consider the following game where mixed strategies are allowed. Assume $a > 0$.

	L	R
T	a,3	a,1
M	2,1	0,0
B	0,0	1,2

(a) **(15 pts)** What is the range of values of a for which T is strictly dominated?

(b) **(15 pts)** Assume that T is strictly dominated. Find the set of all pure and mixed Nash equilibria.

Q3. **(30 pts)** A buyer and a seller of an object are trying to agree on the terms of a trade. The buyer and the seller simultaneously submit prices $p_b \geq 0, p_s \geq 0$, respectively. If $p_b \geq p_s$, trade occurs: the buyer pays p_b, the seller receives p_s, and the difference $p_b - p_s$ goes to a charity. If trade occurs, the buyer's payoff is his valuation of the object, v, minus the price he pays. The seller's payoff is the price received minus his cost, c. Assume $v > c \geq 0$. If trade does not occur, both players receive a payoff of 0.

(a) **(10 pts)** Formulate this situation as a strategic form game.

(b) **(10 pts)** Show that any (p_b, p_s) such that $p_b = p_s$ and $c \leq p_b \leq v$ is a Nash equilibrium.

(c) **(10 pts)** Find the set of all pure strategy Nash equilibria.

Q4. **(30 pts)** Consider this model of a market that combines features of the Hotelling and Bertrand models. Buyers of a good are uniformly located on a line, starting at position 0 at the left and ending at position 1 on the right. There are two stores, A and B, located at position $\frac{1}{4}$ and 1, respectively. Each store charges its own price p_A and p_B. A buyer must choose which store to buy from. The total cost to the buyer of going to a store is the distance to the store's position, plus the price charged by the store. Buyers always choose the store with the lowest total cost.

(a) **(5 pts)** Suppose a buyer is at position x, with $0 \leq x \leq 1$. Write down the buyer's total cost if he chooses store A, and if he chooses store B.

(b) **(5 pts)** At what position x^* is the buyer indifferent between A and B?

(c) **(10 pts)** The length of the line that chooses store A is the quantity demanded from A (similarly for B). Write down the demand for A and B, as a function of p_A, p_B.

(d) **(10 pts)** Assume that costs are zero for both stores, and they each choose their own price to maximize profits. Find the set of Nash equilibrium (if any).