CUR 412: Game Theory and its Applications

Midterm Exam

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Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 90 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(10 pts)** For the following questions, give an example, in matrix form, of a 2-player, 2-action game.

(a) **(5 pts)** Give an example of a game with no pure strategy Nash equilibria.

(b) **(5 pts)** Give an example of a symmetric game with exactly one pure strategy Nash equilibrium.

Q2. **(30 pts)**

	L	C	R
T	4,2	3,0	1,1
M	1,2	2,4	0,3
B	1,1	4,2	2,4

(a) **(15 pts)** Find the set of pure strategy Nash equilibria.

(b) **(15 pts)** Find the set of mixed strategy Nash equilibria..

Q3. **(30 pts)** Three firms are considering whether to enter a new market. A firm that does not enter gets a payoff of 0. A firm that does choose to enter must pay a cost of 62 and makes revenues of $\frac{150}{n}$, where n is the total number of firms that choose to enter; its payoff is the amount of profits it makes.

(a) **(15 pts)** Find the set of pure strategy Nash equilibria.

(b) **(15 pts)** Find the symmetric mixed strategy Nash equilibrium, where all three firms enter with the same probability.

Q4. **(30 pts)** Two firms are competing in a market and their products are imperfect substitutes for one another. The demand functions for the products of firm 1 and 2 are given by:

$$q_1(p_1, p_2) = \begin{cases} a - bp_1 + p_2 & \text{if } a - bp_1 + p_2 \geq 0 \\ 0 & \text{if } a - bp_1 + p_2 < 0 \end{cases}$$

$$q_2(p_1, p_2) = \begin{cases} a - bp_2 + p_1 & \text{if } a - bp_2 + p_1 \geq 0 \\ 0 & \text{if } a - bp_2 + p_1 < 0 \end{cases}$$

where p_1 and p_2 are the prices charged by firm 1 and firm 2, respectively. Prices must be non-negative, and assume that $a > 0$ and $b > 1$. Assume that costs are zero, so profit for firm i is $p_i q_i(p_1, p_2)$.

(a) **(15 pts)** Formulate this situation as a strategic form game.

(b) **(15 pts)** Find the set of all pure strategy Nash equilibria.