

CUR 412: Game Theory and its Applications

Midterm Exam

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Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 90 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(10 pts)** For the following questions, give an example, in matrix form, of a 2-player, 2-action game.

- (a) **(5 pts)** Give an example of a game where the NE are not Pareto-optimal (that is, there is a non-NE outcome in which both players are better off).
- (b) **(5 pts)** Give an example of a symmetric game where both players prefer to play different actions from each other.

Q2. **(30 pts)**

	<i>L</i>	<i>R</i>
<i>T</i>	3,2	1,1
<i>M</i>	2,0	0,0
<i>B</i>	1,1	2,3

- (a) **(15 pts)** Find all dominated actions. State if it is strictly or weakly dominated.
- (b) **(15 pts)** Find all pure and mixed Nash equilibria.

Q3. **(30 pts)** A citizen (player 1) must choose whether to file taxes honestly, or to cheat. The tax collector (player 2) decides how much effort to invest in auditing; he chooses a real number a between 0 and 1, which costs the tax collector $100a^2$. If the citizen is honest, he gets a payoff of 0 and the tax collector gets a payoff of $-100a^2$. If the citizen cheats, then a is the probability of getting caught. If the citizen cheats, the payoffs are:

- If the citizen is caught (with probability a), he gets a payoff of -100 and the tax collector gets a payoff of $100 - 100a^2$.

- If the citizen is not caught (with probability $1 - a$), he gets a payoff of 50 and the tax collector gets a payoff of $-100a^2$.
- (a) **(5 pts)** Suppose the tax collector believes the citizen is honest with probability 1. Find the best response level of a .
- (b) **(5 pts)** Suppose the tax collector believes the citizen is cheating with probability 1. Find the best response level of a .
- (c) **(10 pts)** Suppose the tax collector believes the citizen is cheating with probability p . Find the best response level of a as a function of p .
- (d) **(10 pts)** Find all pure strategy NE, or show that none exists.

Q4. **(30 pts)** Consider the Bertrand model of oligopoly. Suppose there are two firms with the same unit cost c ; each firm chooses a price p_1, p_2 . Buyers only buy from the firm offering the lowest price; if both firms offer the same price, then market demand is split equally. Market demand at price p is given by $D(p) = \alpha - 2p$ for $2p \leq \alpha$, $D(p) = 0$ for $2p > \alpha$. Assume $c < \alpha/2$.

- (a) **(10 pts)** Find the Nash equilibrium prices, output levels, and profits for each firm.
- (b) **(10 pts)** Suppose both firms combined to form a monopoly. Find the equilibrium price, output level, and profit.
- (c) **(10 pts)** Now, suppose that both firms engage in *price matching*: if a customer can find a lower price elsewhere, the firm will reduce their price to match it. Assume that if firms choose (p_1, p_2) , the price that both firms end up facing is $\min(p_1, p_2)$. Write down the payoff function, and show that it is a NE when both firms charge the monopoly price.