CUR 412: Game Theory and its Applications

Midterm Exam

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Instructions:

- Please write your name in English.
- This exam is closed-book.
- Total time: 90 minutes.
- There are 4 questions, for a total of 100 points.

Q1. **(10 pts)** For the following questions, give an example, in matrix form, of a 2-player, 2-action game.

(a) **(5 pts)** Give an example of a game with a (strictly) dominant strategy equilibrium.

(b) **(5 pts)** Give an example of a symmetric game with a Nash equilibrium that is not symmetric.

Q2. **(25 pts)**

	L	C	R
T	3,2	4,0	1,1
M	2,0	3,3	0,0
B	1,1	0,2	2,3

(a) **(10 pts)** Find the set of outcomes that remain after iterated elimination of strictly dominated strategies.

(b) **(15 pts)** Find all pure and mixed Nash equilibria.

Q3. **(25 pts)** Consider the Bertrand oligopoly game with two firms. Firm 1's cost function is $c_1(q_1) = q_1$, and Firm 2's cost function is $c_2(q_2) = 2q_2$. Total demand is given by $P = 100 - Q$, where $Q = q_1 + q_2$. Firms choose prices p_1, p_2 respectively, where $p_1, p_2 \geq 0$, and prices must be an integer multiple of 0.01 (so 0.01, 0.02, 0.03, ... are valid prices). Buyers only buy from the firm offering the lowest price; if there is a tie, both firms split the market equally.

(a) **(5 pts)** Write down the profit functions of both firms.

(b) **(20 pts)** Find the set of Nash equilibria.

Q4. **(40 pts)** Consider a sealed-bid, *all-pay* auction (that is, all bidders pay the same cost, whether they win or not). This can be used to model situations such as government lobbying, where all bidders have to spend resources in order to have a chance at winning. Suppose there are two bidders with valuations v_1, v_2 respectively, and assume that $v_1 > v_2 > 0$. Let x_1, x_2 denote each player's bid, where $x_1 \geq 0, x_2 \geq 0$. If there is a tie, assume Player 1 wins.

Suppose the auction is *second-price* (the highest bidder wins, and both players pay the second highest bid).

(a) **(5 pts)** Write down the payoff functions for both players.

(b) **(15 pts)** Find the set of Nash equilibria.

Now, suppose the auction is *first-price* (the highest bidder wins, and both players pay the highest bid).

(c) **(5 pts)** Write down the payoff functions for both players.

(d) **(15 pts)** Find the set of Nash equilibria.