1 Solutions to Homework 3

1.1 163.1 (Nash equilibria of extensive games)

The strategic form of the game in Exercise 156.2*a* is given in Figure 33.1.

	EG	*EH*	*FG*	*FH*
C	1,0	1,0	3,2	3,2
D	2,3	0,1	2,3	0,1

Figure 33.1 The strategic form of the game in Exercise 156.2*a*.

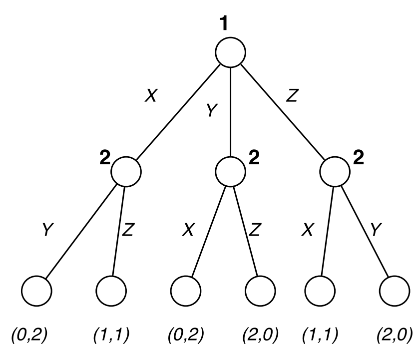
The Nash equilibria of the game are (C, FG), (C, FH), and (D, EG).
The strategic form of the game in Figure 160.1 is given in Figure 33.2.

	E	*F*
CG	1,2	3,1
CH	0,0	3,1
DG	2,0	2,0
DH	2,0	2,0

Figure 33.2 The strategic form of the game in Figure 160.1.

The Nash equilibria of the game are (CH, F), (DG, E), and (DH, E).

1.2 163.2 (Voting by Alternating Veto)

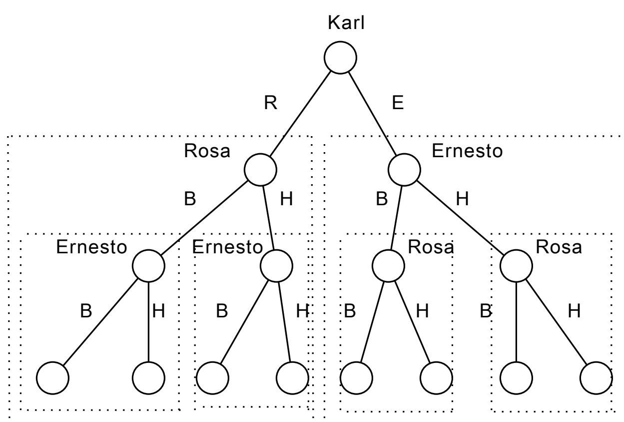


Let the action of each player be the policy that he vetoes. The strategic form of this game is given by this matrix (Player 2 is the row player):

	X	Y	Z
YXX	0,2	0,2	1,1
YXY	0,2	0,2	2,0
YZX	0,2	2,0	1,1
YZY	0,2	2,0	2,0
ZXX	1,1	0,2	1,1
ZXY	1,1	0,2	2,0
ZZX	1,1	2,0	1,1
ZZY	1,1	2,0	2,0

The NE are $(Z, YXX), (Z, ZXX)$.

1.3 164.2 (Subgames)



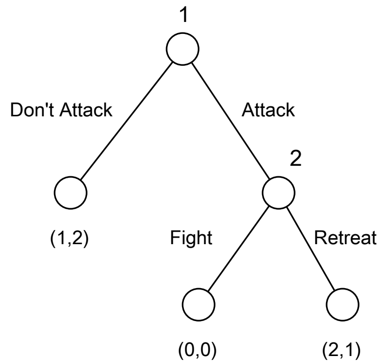
There are 6 proper subgames, beginning at every node where Rosa or Ernesto chooses an action.

1.4 173.3 (Voting by Alternating Veto)

The unique SPNE is (Z, YXX). The other NE is not a SPNE, since there is an incredible threat that Player 2 will veto Z if Player 1 vetoes X. The outcome ZX is the same in both NE.

If we change preferences such that Player 1 prefers $Y > X > Z$, then the SPNE outcome will be different depending on which player goes first.

1.5 173.4 (Burning a bridge)



Note that this has the same structure as the Entry Game. The SPNE is $(Attack, Retreat)$. If it were possible for Army 2 to burn a bridge (i.e. remove the $Retreat$ action), then the optimal action in the subgame would be $Fight$, and the SPNE would be $(Don'tAttack, Fight)$ which has a higher payoff for Army 2.

1.6 183.2 (Subgame perfect equilibria of the ultimatum game with indivisible units)

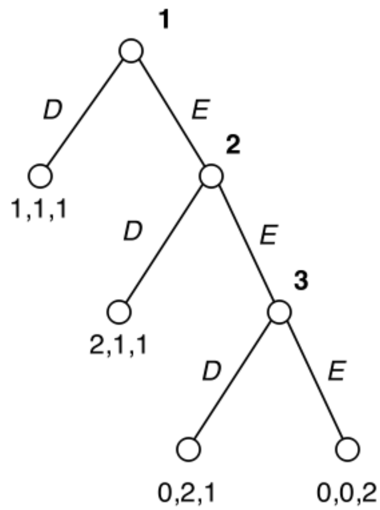
Suppose that $x \leq c$ is the offer, and offers must be integer multiples of 1 cent. In the second stage, Player 2 has 2 possible strategies:

1. Accept if $x \geq 0$. Then Player 1's optimal choice is to offer $x = 0$.
2. Accept if $x \geq 1$ cent Then Player 1's optimal choice is to offer $x = 1$ cent.

There are two SPNE:

1. Player 1 offers $x = 0$, and Player 2's strategy is: accept if $x \geq 0$.
2. Player 1 offers $x = 1$ cent, and Player 2's strategy is: accept if $x \geq 1$ cent.

1.7 202.1 (Hungry lions)



The tree for 3 lions is shown above. Let the actions be D for Don't Eat, and E for Eat. Suppose you are the n-th lion. If the $(n+1)$-th lion chooses E, then your optimal choice is D. If the $(n+1)$-th lion chooses D, then your optimal choice is E. The last lion will choose E, the second to last lion will choose D, then E, D, E... in an alternating fashion. If there are an even number of lions, the first lion will choose D, otherwise it will choose E.