CUR 412: Game Theory and its Applications, Lecture 4

Prof. Ronaldo CARPIO

March 18, 2013

Hotelling's Model of Electoral Competition

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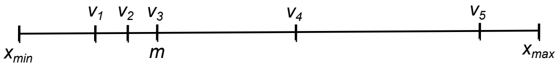
- Here, location represents a position on a *one-dimensional* political spectrum, but it can also represent physical space or product space.

Location on Political Spectrum



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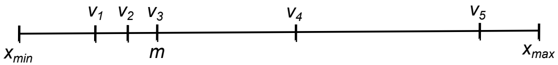

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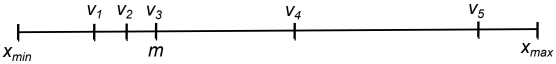

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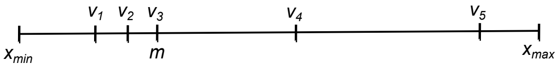

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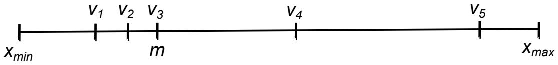

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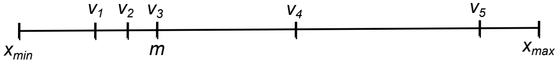

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- In this example, there are five voters with favorite positions at $v_1 \ldots v_5$.
- The *median* position m is the position such that half of voters are to the left or equal to m, and the other half are to the right or equal to m.
- Voters dislike positions that are farther away from them on the line. They are indifferent between positions to their left and right that have the same distance.

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- Therefore, each candidate will attract all voters who are closer to him than any other candidate.

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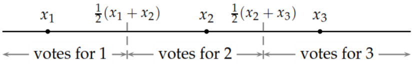

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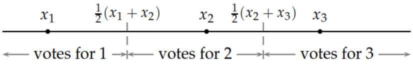

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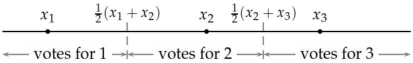

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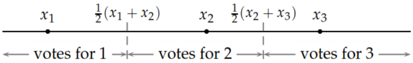

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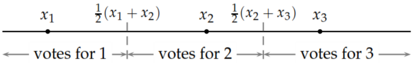

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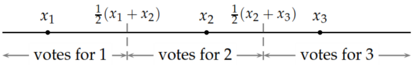

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- Losing is the least preferable outcome.

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  - Preferences: Each candidate's payoff is given by the function above.

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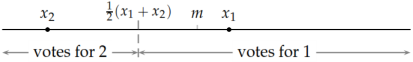

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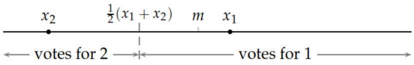

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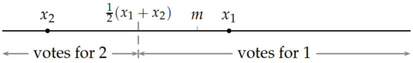

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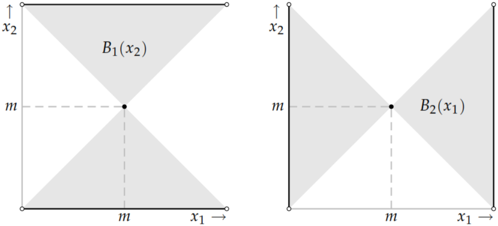
- Case 2: $x_2 > m$
  - By the same reasoning, every position between $2m - x_j$ and x_j is a best response.
- Case 3: $x_2 = m$
  - Choosing m results in a tie; any other choice results in a loss. Therefore, $x_1 = m$ is the best response.

Best Response Function



- Best response function is:

$$B_1(x_2) = \begin{cases} \{x_1 : x_2 < x1 < 2m - x2\} & \text{if } x_2 < m \\ \{m\} & \text{if } x_2 = m \\ \{x_1 : 2m - x_2 < x1 < x2\} & \text{if } x_2 > m \end{cases}$$

- Unique Nash equilibrium is when both candidates choose m.

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- Requires the one-dimensional assumption on voter/consumer preferences.
- If there is more than one dimension (e.g. consumers care about both price and quality), this result may not hold

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- This can be applied to any kind of dispute between parties, where there is some cost to waiting.

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- Time is costly. For each unit of time that passes before one side concedes, both players lose 1 in payoff.

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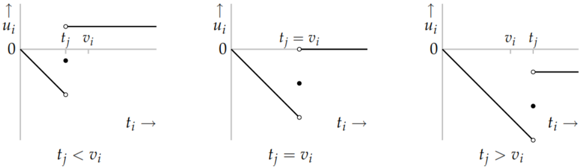

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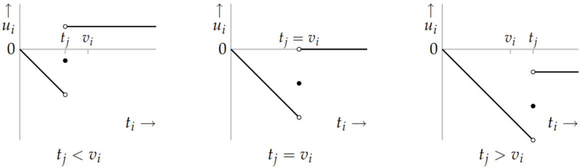

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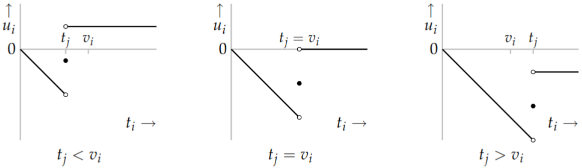

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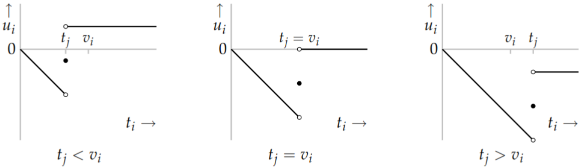

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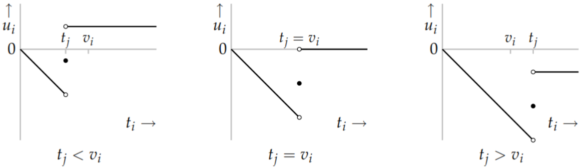

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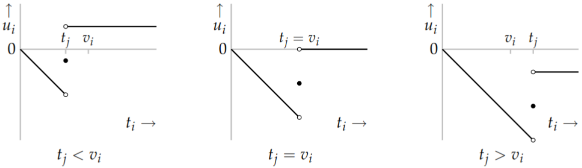

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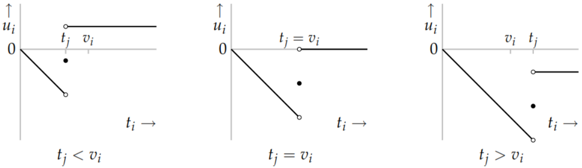

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  - $t_i = 0$ is the best response. Payoff: 0

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- Equilibria are asymmetric: each player chooses a different action, even if they have the same value
- This can only be a stable social norm if players come from different populations (e.g. owners always concede, challengers always wait)

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- The actual winning bid has to only be slightly higher than the second-highest bid.
- We can model this as a *second-price* auction: the winner is the highest bidder, but only has to pay the second-highest price.

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- To break ties, assume player with the highest valuation wins.

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- There are many Nash equilibria in this game. Let's examine some special cases.

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- Players 2 ... n:
  - If Player i lowers bid, still loses.
  - If Player i raises bid to above $b_1 = v_1$, wins, but gets negative payoff $v_i - v_1 < 0$

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  - Any change in bid results in same outcome (because of tie-breaking rules)
- Players 2 ... n:
  - If Player i raises bid to $\leq v_1$, still loses
  - If Player i raises bid to $> v_1$, wins, but gets negative payoff $v_i - v_1$
- This outcome is better off for player 1, but worse off for the seller of the object