Intermediate Microeconomics

Solutions to Midterm Exam

Q1. **(15 pts)** Answer True or False. You don't have to give an explanation.

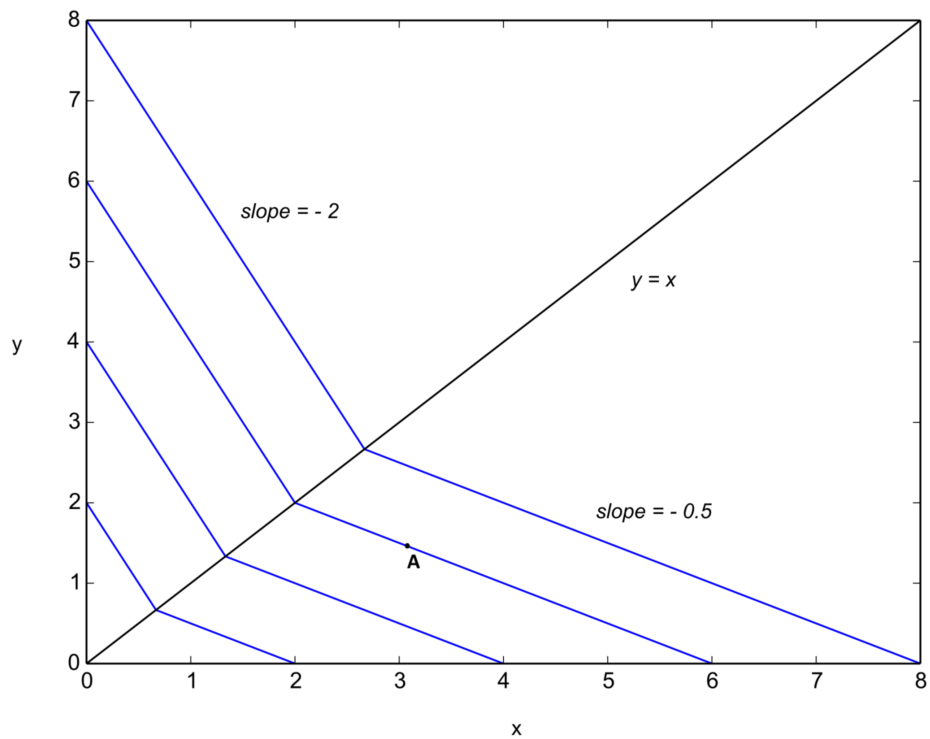
(a) Someone with decreasing marginal utility of wealth is risk-loving.

False. A utility function with decreasing marginal utility is concave, and therefore risk-averse.

(b) If a consumer choosing from two goods is always spending all his money, it is possible for both goods to be inferior.

False. An inferior good is one in which demand decreases as income increases; if demand for both goods goes down as income increases, then the consumer cannot be spending all his money.

For the next 3 questions, refer to the diagram below.

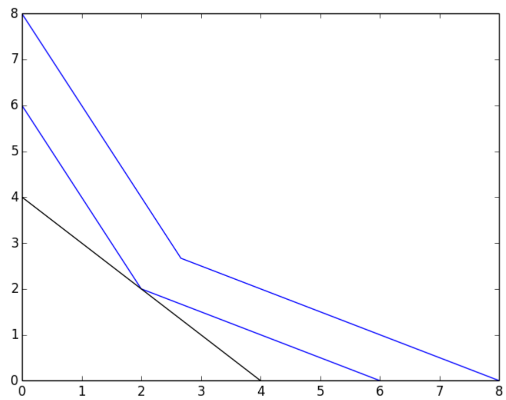


Suppose a consumer choosing from two goods x and y has indifference curves as shown in the figure above. The sharp corner in the indifference curve occurs when $x = y$. If $x > y$, the indifference curve has a slope of -0.5; if $x < y$, it has a slope of -2. Suppose the prices of goods x, y are p_x, p_y respectively.

Recall that the the consumer will choose the highest indifference curve that touches the budget line. If the slope of the budget line is:

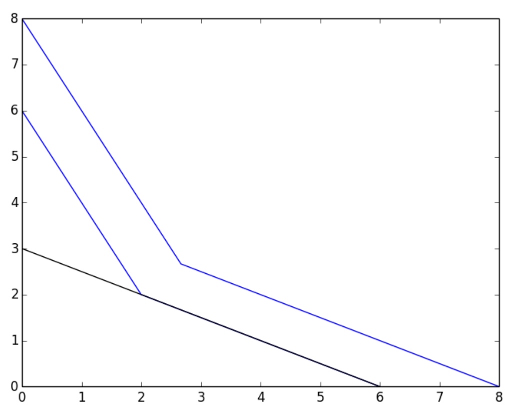
1. between (and not including) 1/2 and 2, then the corner at the midpoint of the indifference curve will be the only point of contact with the budget line, and therefore the consumer will choose $x = y$.

Figure 1: Case 1



2. equal to 1/2, then the lower half of the indifference curve coincides with the budget line, so any point on the lower half is an optimal choice for the consumer.

Figure 2: Case 2



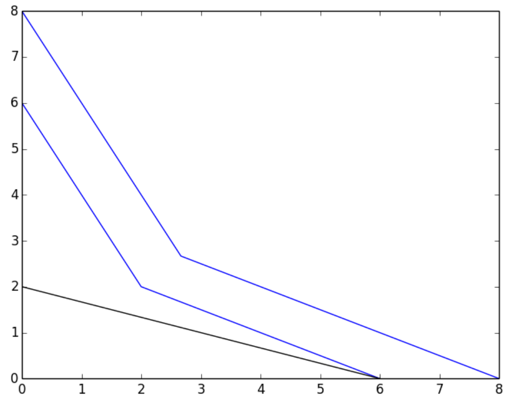
3. equal to 2, then the upper half of the indifference curve coincides with the budget line, so any point on the upper half is an optimal choice for the consumer.

4. less than 0.5, then the point of contact will be at the bottom, where $y = 0$ and all income is spent on good x.

5. greater than 2, then the point of contact will be at the top, where $x = 0$ and all income is spent on good y.

(d) If $p_x = 1/2$ and $p_y = 3$, the consumer will choose $x = 0$.

Figure 3: Case 4



False. The slope of the budget line is less than 0.5, so the consumer will choose $y = 0$.

(e) If $p_x = 2$ and $p_y = 3$, the consumer will choose a point where $x = y$.

True. The slope of the budget line is between 0.5 and 2.

(f) There is no p_x, p_y such that choosing point A is optimal.

False. If $p_x/p_y = 0.5$, the all points on the lower half of the indifference curve are optimal, including A.

Q2. **(40 pts)** A consumer's utility function over wealth is $\sqrt{w}$. Suppose this consumer is betting on a sporting event. If his team wins (with probability $0 \leq q \leq 1$), his wealth is w_A. If it loses (with probability $1 - q$), his wealth is w_B.

(a) **(5 pts)** Write down the formula for his expected utility, as a function of q, w_A, and w_B.

There are two possible outcomes: with probability q, his wealth will be w_A (giving a utility of $\sqrt{w_A}$), and with probability $1 - q$, his wealth will be w_B (giving a utility of $\sqrt{w_B}$). Therefore, expected utility is:

$$EU(q, w_A, w_B) = q\sqrt{w_A} + (1 - q)\sqrt{w_B}$$

(b) **(5 pts)** Is this consumer risk-averse? How can you prove it?

$\sqrt{w}$ is concave, therefore the consumer is risk averse. This can be verified by checking that the derivative, $\frac{1}{2\sqrt{w}}$, is a decreasing function.

(c) **(10 pts)** Suppose $q = 1/2, w_A = 4$, and $w_B = 9$. What *certain* (i.e. non-random) amount of wealth gives the same expected utility? Find the risk premium.

Expected utility is $\frac{1}{2}\sqrt{4} + \frac{1}{2}\sqrt{9} = \frac{5}{2}$. Suppose x gives the same amount of expected utility; then $\sqrt{x} = \frac{5}{2}$, and $x = \frac{25}{4}$. The risk premium is the difference between x and the expected wealth, which is $\frac{1}{2}4 + \frac{1}{2}9 = \frac{13}{2}$. Therefore, the risk premium is $\frac{1}{4}$.

Now, we can interpret w_A and w_B as quantities of *goods*. w_A is the quantity of a lottery ticket that pays \$1 only if the consumer's team *wins*; similarly, w_B is the quantity of a ticket that pays \$1 only if the consumer's team *loses*. (In economics, these are called *contingent claims*). The consumer's utility function over w_A, w_B is the expected utility in part (a).

(e) **(5 pts)** Write down the MRS as a function of q, w_A, and w_B.

$$MRS = \frac{MU_{w_A}}{MU_{w_B}} = \frac{qw_A^{-\frac{1}{2}}}{(1-q)w_B^{-\frac{1}{2}}} = \frac{(q)w_B^{\frac{1}{2}}}{(1-q)w_A^{\frac{1}{2}}}$$

(f) **(5 pts)** Suppose the prices of goods w_A, w_B are p_A, p_B respectively. What is the condition for optimality?

The condition for optimality is that MRS $= \frac{p_A}{p_B}$, or

$$\frac{qw_B^{\frac{1}{2}}}{(1-q)w_A^{\frac{1}{2}}} = \frac{p_A}{p_B}$$

(g) **(10 pts)** Suppose $q = 1/2, p_A = 2, p_B = 2$, and the consumer's income is 4. Find the optimal amounts of w_A and w_B.

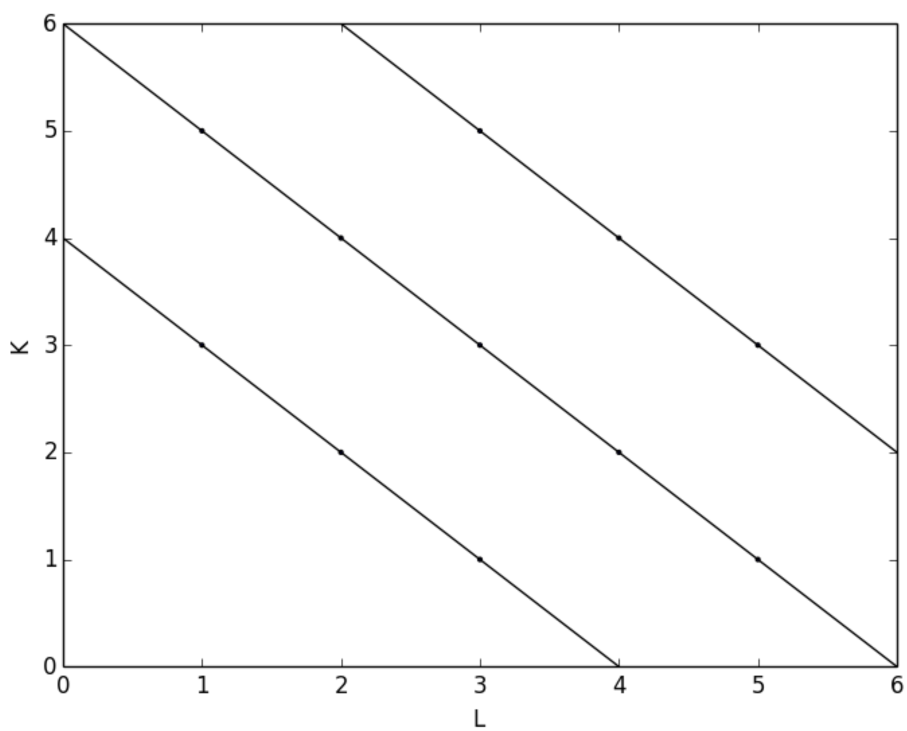
Given these parameters, then $w_B = w_A$. Since the price for both goods is the same, then the amount spent on each good is the same, which must be 2 for each; therefore, $w_A = w_B = 1$.

Q3. **(25 pts)**

		Labor (L)				
		1	2	3	4	5
Capital (K)	1	35	60	70	85	95
	2	60	70	85	95	105
	3	70	85	95	105	115
	4	85	95	105	115	125
	5	95	105	115	125	135

Suppose you are given the table above for a production process with two variable inputs.

(a) **(10 pts)** Plot the isoquants for the output levels 70, 95, and 115.



(b) **(10 pts)** What property does the MRTS satisfy? What is the relationship between the inputs?

The MRTS is constant, since the isoquants are linear. Therefore, the inputs are perfect subsitutes.

(c) **(5 pts)** What kind of returns to scale does this production function have?

This production function has decreasing returns to scale, since if input is doubled, output is less than doubled (for example, consider $K = 2, L = 2$ with an output of 70, compared to $K = 4, L = 4$ with an output of 115).

Q4. **(20 pts)** Suppose an investor is considering a business opportunity that costs 100, and generates a random return according to the probability distribution given below.

Probability	Payoff
0.2	100
0.3	30
0.2	-10
0.3	-30

(a) **(15 pts)** Find the expected return, standard deviation, and variance of the investment.

The expected return is $0.2(100) + 0.3(30) + 0.2(-10) + 0.3(-30) = 18$. The variance is $0.2(100 - 18)^2 + 0.3(30 - 18)^2 + 0.2(-10 - 18)^2 + 0.3(-30 - 18)^2 = 2236$. The standard deviation is the square root of the variance, which is 47.286.

(b) **(2 pts)** Would a risk-neutral investor take this opportunity, if there is no alternative investment?

No. The total expected return would be 18 - 100, which is negative. The investor would be better off simply holding on to the 100.

(c) **(3 pts)** Would a risk-averse investor take this opportunity?

No. For a risk-averse investor, a potential investment that offered lower risk and a lower expected return might be acceptable; but this investment offers both a lower expected return, and higher risk, compared to simply doing nothing and holding on to the 100. So a risk-averse investor will also avoid this investment.