Intermediate Microeconomics

Final Exam

Ronaldo Carpio Dec. 14, 2015

Instructions:

- This exam is closed-book.
- Total time: 2 hours.
- There are 5 questions, for a total of 100 points.

Q1. **(15 pts)** Answer True or False. You don't have to give an explanation.

(a) A perfectly inelastic demand curve is horizontal.

False. A horizontal demand curve implies that demand goes to zero if there is a small price increase, i.e. demand is perfectly *elastic*.

(b) Variable cost is zero when output is zero.

True. A variable cost depends on the output quantity, so if there is a cost when output is zero, it must be a fixed cost.

(c) If average fixed cost is falling, then marginal cost must be rising.

False. FC is a constant, so $AFC = FC/q$ is falling by definition.

(d) Suppose we have a 2-player, 2-action game in matrix form. If we add 10 to Player 1's payoff in every cell, the set of Nash equilibria may change.

False. The preference ranking on outcomes of Player 1 is unchanged if all his payoffs are changed by the same number, so the set of Nash equilibria does not change.

(e) Every game has at least one Nash equilibrium.

False. For example, the game Matching Pennies does not have a Nash equilibrium.

Q2. **(20 pts)** Suppose a firm in a perfectly competitive industry has a cost function of $C(q) = 4q^2 + 16$.

(a) **(5 pts)** Write down the firm's fixed cost, variable cost, average cost, and marginal cost as functions of q.

$FC(q) = 16$, $VC(q) = 4q^2$, $ATC(q) = (4q^2 + 16)/q = 4q + 16/q$, $MC(q) = 8q$.

(b) **(5 pts)** Find the output that minimizes average cost.

The derivative of $ATC(q)$ is $4 - \frac{16}{q^2}$. Setting this to zero and solving for q gives $q = 2$.

(c) **(5 pts)** What is the price at which the firm will choose to produce zero output?

This occurs at the minimum of $AVC(q) = 4q$. This is a linear function and is only minimized at $q = 0$, where $AVC(0) = 0$. Therefore, the firm will produce positive output at any positive price, and produce zero output only when the price is zero.

(d) **(5 pts)** What is the price at which the firm makes zero profit?

This occurs at the minimum of $ATC(q)$. We can plug in the answer to part (b) into $ATC(q)$ to get 16.

Q3. **(30 pts)** Suppose there is a market with an inverse demand function given by $P(Q) = 2000 - 2Q$, where Q is the total quantity produced by all firms.

(a) **(5 pts)** Suppose there is a single firm that behaves as a price-taker, with a total cost function of $C(q) = 80000 + 560q$. Find the profit-maximizing level of output and the market price.

A price taker's optimality condition is $P = MC$. $MC = 560$, solving $2000 - 2Q = 560$ gives $Q = 720, P = 560$.

(b) **(10 pts)** Suppose the same firm in (a) now behaves as a monopolist. Find the profit-maximizing level of output and the market price.

Revenue is $Q(2000 - 2Q) = 2000Q - 2Q^2$. MR is $2000 - 4Q$. The optimality condition is $MR = MC$. Solving $2000 - 4Q = 560$ gives $Q = 360, P = 1280$.

(c) **(15 pts)** Now, suppose there are two firms with the same cost function as in (a), engaged in Cournot duopoly. Find the equilibrium output of each firm and the market price.

Consider firm 1's profit:

$$\pi_1(q1, q2) = q_1 P(Q) - C(q_1) = q_1(2000 - 2(q_1 + q_2)) - 80000 - 560q$$
$$= -2q_1^2 + q_1(1440 - 2q_2) - 80000$$

Setting the derivative to zero:

$$1440 - 4q_1 - 2q_2 = 0 \Rightarrow q_1 = \frac{720 - q_2}{2}$$

This is the best response function of firm 1 to q_2. Since both firms are identical, we also have firm 2's best response to q_1:

$$q_2 = \frac{720 - q_1}{2}$$

Solving this system of equations for q_1 and q_2, we get $q_1 = q_2 = 240$. The market price is $2000 - 2(240 + 240) = 1040$.

Note: A common (but erroneous) approach was to do something like this: since both firms are identical, then we can assume that $q_1 = q_2$. Let x denote $q_1 = q_2$. The firm's revenue is $x \cdot P(Q) = x(2000-4x) = 2000x-4x^2$. Marginal revenue is $2000-8x$, and setting $MR = MC$, we get $2000 - 8x = 560 \Rightarrow x = q_1 = q_2 = 180$, $P = 2000 - 4 \cdot 180 = 1280$. There is a subtle (but critical) difference here: when we try to choose the optimal x (as when we set $MR = MC$), we are optimizing for *both* firms simultaneously, rather than one firm optimizing, taking the other's output as given. This is essentially treating the two firms as colluding together to behave like a monopolist, which can be verified by checking Q and P; they are the same as in part (b), the monopolistic case. However, as we have seen, the equilibrium output and prices for a duopoly should be between the perfect competition and monopoly outcomes.

Q4. **(15 pts)** Find the best response functions of both players, and the set of Nash equilibria for the following game.

	L	C	R
T	7,7	4,2	$1,\overline{8}$
M	2,4	$\underline{5},\overline{5}$	$\underline{2}$,3
B	$\underline{8}$,1	$3,\overline{2}$	0,0

The only NE is (M, C).

Q5. **(20 pts)** A firm has a production function given by $q = 5KL$. Suppose the wage rate is $w = 2$ and the cost of capital is $r = 4$.

(a) **(5 pts)** Write down the MRTS of the production function.

MRTS $= \frac{MP_L}{MP_K} = \frac{5K}{5L} = \frac{K}{L}$

(b) **(5 pts)** In the long run, what combination of K, L should the firm choose to produce an output level of 10?

The optimality condition is to set the slope of the isoquant (i.e. the MRTS) to the slope of the isocost line.

$$\frac{K}{L} = \frac{2}{4} = \frac{1}{2} \Rightarrow L = 2K$$

Plugging this into the production function, $10 = 5KL = 5K \cdot 2K$, which gives $K = 1, L = 2$.

(c) **(5 pts)** In the short run, suppose K is fixed at $K = 3$. Find the firm's total cost function as a function of q.

Costs are $wL + rK = 2L + 4 \cdot 3 = 12 + 2L$. The production function gives $q = 5 \cdot 3L$, or $L = q/15$. The cost as a function of q is $12 + \frac{2}{15}q$.

(d) **(5 pts)** Find the short-run fixed cost, variable cost, and marginal cost.

$FC(q) = 12$, $VC(q) = \frac{2}{15}q$, $MC(q) = \frac{2}{15}$.