Intermediate Microeconomics

Solutions to Homework #2

Q1: Chapter 5, Exercise 1 in the textbook.

1. **Consider a lottery with three possible outcomes:**
    - **\$125 will be received with probability .2**
    - **\$100 will be received with probability .3**
    - **\$50 will be received with probability .5**

   a. **What is the expected value of the lottery?**

   The expected value, *EV*, of the lottery is equal to the sum of the returns weighted by their probabilities:

   $$EV = (0.2)(\$125) + (0.3)(\$100) + (0.5)(\$50) = \$80.$$

   b. **What is the variance of the outcomes?**

   The variance, σ^2, is the sum of the squared deviations from the mean, \$80, weighted by their probabilities:

   $$\sigma^2 = (0.2)(125 - 80)^2 + (0.3)(100 - 80)^2 + (0.5)(50 - 80)^2 = \$975.$$

   c. **What would a risk-neutral person pay to play the lottery?**

   A risk-neutral person would pay the expected value of the lottery: \$80.

Q2: Chapter 5, Exercise 6 in the textbook.

6. Suppose that Natasha's utility function is given by $u(I) = \sqrt{10I}$, where I represents annual income in thousands of dollars.

a. Is Natasha risk loving, risk neutral, or risk averse? Explain.

Natasha is risk averse. To show this, assume that she has $10,000 and is offered a gamble of a $1000 gain with 50 percent probability and a $1000 loss with 50 percent probability. Her utility of $10,000 is $u(10) = \sqrt{10(10)} = 10$. Her expected utility with the gamble is:

$$EU = (0.5)\sqrt{10(11)} + (0.5)\sqrt{10(9)} = 9.987 < 10.$$

She would avoid the gamble. If she were risk neutral, she would be indifferent between the $10,000 and the gamble, and if she were risk loving, she would prefer the gamble.

You can also see that she is risk averse by noting that the square root function increases at a decreasing rate (the second derivative is negative), implying diminishing marginal utility.

b. Suppose that Natasha is currently earning an income of \$40,000 (I = 40) and can earn that income next year with certainty. She is offered a chance to take a new job that offers a .6 probability of earning \$44,000 and a .4 probability of earning \$33,000. Should she take the new job?

The utility of her current salary is $\sqrt{10(40)} = 20$. The expected utility of the new job's salary is

$$EU = (0.6)\sqrt{10(44)} + (0.4)\sqrt{10(33)} = 19.85,$$

which is less than 20. Therefore, she should not take the job. You can also determine that Natasha should reject the job by noting that the expected value of the new job is only \$39,600, which is less than her current salary. Since she is risk averse, she should never accept a risky salary with a lower expected value than her current certain salary.

c. In (b), would Natasha be willing to buy insurance to protect against the variable income associated with the new job? If so, how much would she be willing to pay for that insurance? (*Hint*: What is the risk premium?)

This question assumes that Natasha takes the new job (for some unexplained reason). Her expected salary is 0.6(44,000) + 0.4(33,000) = \$39,600. The risk premium is the amount Natasha would be willing to pay so that she receives the expected salary for certain rather than the risky salary in her new job. In part (b) we determined that her new job has an expected utility of 19.85. We need to find the certain salary that gives Natasha the same utility of 19.85, so we want to find I such that $u(I)$ = 19.85. Using her utility function, we want to solve the following equation: $\sqrt{10I} = 19.85$. Squaring both sides, $10I$ =394.02, and I = 39.402. So Natasha would be equally happy with a certain salary of \$39,402 or the uncertain salary with an expected value of \$39,600. Her risk premium is \$39,600 – 39,402 = \$198. Natasha would be willing to pay \$198 to guarantee her income would be \$39,600 for certain and eliminate the risk associated with her new job.

Q3: Chapter 5, Exercise 7 in the textbook.

7. Suppose that two investments have the same three payoffs, but the probability associated with each payoff differs, as illustrated in the table below:

Payoff	**Probability (Investment A)**	**Probability (Investment B)**
$300	**0.10**	**0.30**
$250	**0.80**	**0.40**
$200	**0.10**	**0.30**

a. **Find the expected return and standard deviation of each investment.**

The expected value of the return on investment A is

$$EV = (0.1)(300) + (0.8)(250) + (0.1)(200) = \$250.$$

The variance on investment A is

$$\sigma^2 = (0.1)(300 - 250)^2 + (0.8)(250 - 250)^2 + (0.1)(200 - 250)^2 = \$500,$$

and the standard deviation on investment A is $\sigma = \sqrt{500} = \$22.36$.

The expected value of the return on investment B is

$$EV = (0.3)(300) + (0.4)(250) + (0.3)(200) = \$250.$$

The variance on investment B is

$$\sigma^2 = (0.3)(300 - 250)^2 + (0.4)(250 - 250)^2 + (0.3)(200 - 250)^2 = \$1500,$$

and the standard deviation on investment B is $\sigma = \sqrt{1500} = \$38.73$.

b. **Jill has the utility function $U = 5I$, where I denotes the payoff. Which investment will she choose?**

Jill's expected utility from investment A is

$$EU = (0.1)[5(300)] + (0.8)[5(250)] + (0.1)[5(200)] = 1250.$$

Jill's expected utility from investment B is

$$EU = (0.3)[5(300)] + (0.4)[5(250)] + (0.3)[5(200)] = 1250.$$

Since both investments give Jill the same expected utility she will be indifferent between the two. Note that Jill is risk neutral, so she cares only about expected values. Since investments A and B have the same expected values, she is indifferent between them.

c. Ken has the utility function $U = 5\sqrt{I}$. Which investment will he choose?

Ken's expected utility from investment A is

$$EU = (0.1)(5\sqrt{300}) + (0.8)(5\sqrt{250}) + (0.1)(5\sqrt{200}) = 78.98.$$

Ken's expected utility from investment B is

$$EU=(0.3)(5\sqrt{300}) + (0.4)(5\sqrt{250}) + (0.3)(5\sqrt{200}) = 78.82.$$

Ken will choose investment A because it has a slightly higher expected utility. Notice that Ken is risk averse, so he prefers the investment with less variability.

d. Laura has the utility function $U = 5I^2$. Which investment will she choose?

Laura's expected utility from investment A is

$$EU=(0.1)[5(300^2)] + (0.8)[5(250^2)] + (0.1)[5(200^2)] = 315{,}000.$$

Laura's expected utility from investment B is

$$EU=(0.3)[5(300^2)] + (0.4)[5(250^2)] + (0.3)[5(200^2)] = 320{,}000.$$

Laura will choose investment B since it has a higher expected utility. Notice that Laura is a risk lover, so she prefers the investment with greater variability.

Q4: Chapter 6, Exercise 3 in the textbook.

3. Fill in the gaps in the table below.

Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	–	–
1	225		
2			300
3		300	
4	1140		
5		225	
6			225

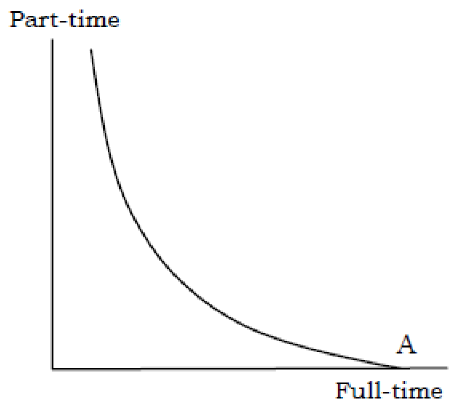
Quantity of Variable Input	Total Output	Marginal Product of Variable Input	Average Product of Variable Input
0	0	___	___
1	225	225	225
2	600	375	300
3	900	300	300
4	1140	240	285
5	1365	225	273
6	1350	–15	225

Q5: Chapter 6, Exercise 5 in the textbook.

5. For each of the following examples, draw a representative isoquant. What can you say about the marginal rate of technical substitution in each case?

a. A firm can hire only full-time employees to produce its output, or it can hire some combination of full-time and part-time employees. For each full-time worker let go, the firm must hire an increasing number of temporary employees to maintain the same level of output.

Place part-time workers on the vertical axis and full-time workers on the horizontal. The slope of the isoquant measures the number of part-time workers that can be exchanged for a full-time worker while still maintaining output. At the bottom end of the isoquant, at point A, the isoquant hits the full-time axis because it is possible to produce with full-time workers only and no part-timers. As we move up the isoquant and give up full-time workers, we must hire more and more part-time workers to replace each full-time worker. The slope increases (in absolute value) as we move up the isoquant. The isoquant is therefore convex and there is a diminishing marginal rate of technical substitution.



b. A firm finds that it can always trade two units of labor for one unit of capital and still keep output constant.

The marginal rate of technical substitution measures the number of units of capital that can be exchanged for a unit of labor while still maintaining output. If the firm can always trade two units of labor for one unit of capital then the MRTS of labor for capital is constant and equal to 1/2, and the isoquant is linear.

c. A firm requires exactly two full-time workers to operate each piece of machinery in the factory

This firm operates under a fixed proportions technology, and the isoquants are L-shaped. The firm cannot substitute any labor for capital and still maintain output because it must maintain a fixed 2:1 ratio of labor to capital. The MRTS is infinite (or undefined) along the vertical part of the isoquant and zero on the horizontal part.