# Microeconomics of Banking: Lecture 3

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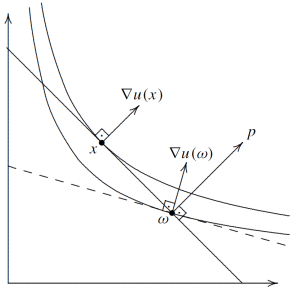
# Review of Last Week

- Consumer choice problem
- General equilibrium
- Contingent claims
- Risk aversion




- The optimal choice, $x = (X, Y)$, is where the indifference curve is tangent to the budget line.
- The slope of the indifference curve (= MRS) is equal to the slope of the budget line (= the price ratio).
- Another way to state this is $\nabla U = \lambda(P_x, P_y)^T$: the gradient of the utility function is a scalar multiple of the price vector
- Suppose the consumer has an initial endowment of goods $\omega = (X_\omega, Y_\omega)$. Then this is equivalent to having an income of $I = P_x X_\omega + P_y Y_\omega$.

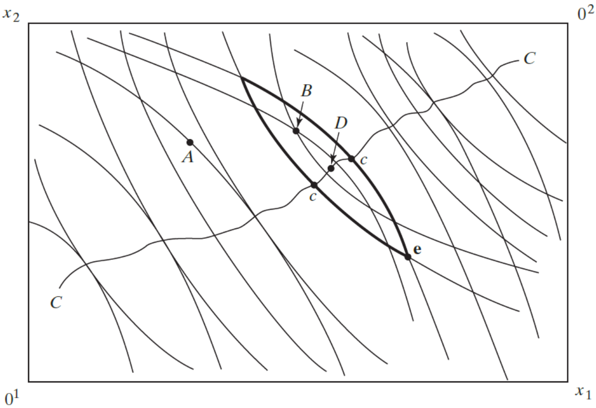



**Figure 5.2.** Equilibrium in two-person exchange.

- An allocation on the Edgeworth box is a Walrasian equilibrium if neither agent has any incentive to trade.
- This occurs when the indifference curves of both agents are tangent to each other.

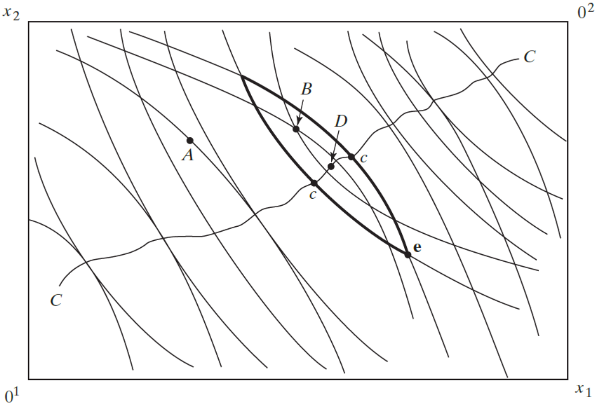



**Figure 5.2.** Equilibrium in two-person exchange.

- The price vector is then the slope of the indifference curves at the equilibrium allocation point.
- You can think of this as two consumer choice problems being solved simultaneously, using the same prices.

# Contingent Claims

- We can extend consumer choice framework to handle time and risk by expanding our definition of what a *good* is.
- We can add the *time of availability* to the definition of a good.
- For example: instead of two goods "apple" and "orange", we can define the good "1 apple today" and "1 orange, one week from now".
- We can also define a good to be *contingent on a random event*.
- Examples:
  - "umbrella when it is raining" vs. "umbrella when it is not raining"
  - "1 unit of grain when the harvest is good" vs. "1 unit of grain when the harvest is bad"
  - "income when there is a recession" vs. "income when there is an expansion"
- These are called *contingent claims*.

# Two-Period Model

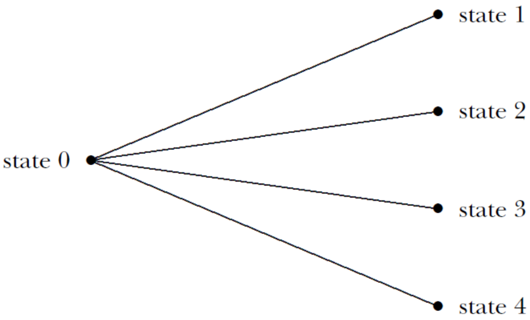



***Figure 2.2.*** *A simple state tree.*

- At $t = 0$, there is complete uncertainty: the only information agents have is that all states of the world are possible.
- At $t = 1$, uncertainty is resolved: all agents know exactly which state of the world has occurred.

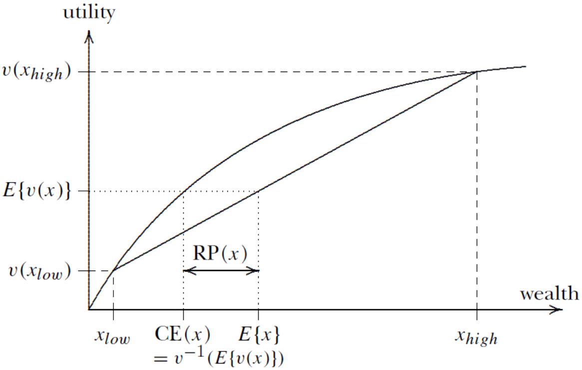



***Figure 4.8.*** *A risk-averse NM utility is concave.*

- Let $E(x) = \pi_{low} x_{low} + \pi_{high} x_{high}$, the expected value of $x$.
- A risk-averse consumer gets higher expected utility from the lottery $(E(x), E(x))$ than from $(x_{low}, x_{high})$.

# Finance Economy

- A *finance economy* combines the GE model with risk averse utility functions.
- Suppose there is only one consumption good, call it money, income or wealth.
- A *financial asset* is a security that entitles its holder to a specified payout for each possible state of the world.
- Suppose that asset $j$ is specified by: $r^j = (r_1^j, ..., r_S^j)^T$
- Whoever holds 1 unit of asset $j$ will receive $r_s^j$ at $t = 1$, if the state of the world happens to be $s$.
- $r_s^j$ is called a *state-contingent payoff*.

- The car insurance in last lecture's example could be thought of as having two state-contingent payoffs:
  - in the *crash* state, pays 1
  - in the *ok* state, pays 0
- An asset is called "risky" if it gives a different payoff in different states.
- An asset is called "riskless" or "risk-free" if it gives the same payoff in every state.
- A *storage asset* (e.g. cash) would be $(1, ..., 1)^T$.
- A *riskless bond* with nominal yield $1 + r$ would be $(1 + r, ..., 1 + r)^T$.

# Finance Economy

- In the real world, stocks or equities are a claim of ownership over a fraction of a firm.
- This entitles the stockholder to a fraction of the profits of the firm.
- A bond is a loan, which may or may not be repaid.
- In the model, stocks and non-government bonds are modeled as *risky* assets, that is, their payouts will be different in different states of the world.

# Finance Economy

- For example, if the possible states of the world are *recession* and *expansion*, then a stock could give a high payout in the *expansion* state and a low payout in the *recession* state.
- For bonds, the states of the world might be *success* or *bankrupt*. The bond gives the promised payout in the *success* state, and gives a zero payout in the *bankrupt* state.
- A *riskless* bond is a special kind of bond, that gives the same payout in all states.
- Obviously nothing is completely riskless in real life, but for our purposes we can treat the sovereign debt of rich nations with no history of default as "riskless".

$$r^j = \begin{bmatrix} r_1^j \\ r_2^j \\ \vdots \\ r_S^j \end{bmatrix}$$

- We can write down the *state-contingent payoffs* of a particular asset in a vector.

$$
\begin{array}{c c}
 & \textit{securities} \\
 & \begin{array}{ccc} 1 & \cdots & J \end{array} \\
\rotatebox{90}{\textit{states}} \begin{array}{c} 1 \\ \vdots \\ S \end{array} & \left[ \begin{array}{ccc} r_1^1 & \cdots & r_1^J \\ \vdots & \ddots & \vdots \\ r_S^1 & \cdots & r_S^J \end{array} \right] =: r.
\end{array}
$$

- Suppose there are $J$ assets.
- We can write all their state-contingent payoffs in a matrix.

- Suppose we have a square $n \times n$ matrix $M$.
- A row (or column) of $M$ is called *linearly independent* if it cannot be written as a linear combination of the other rows (or columns).
- A basic fact from linear algebra is that if $M$ has $n$ linearly independent rows (or columns), then any $n$-dimensional vector can be written as a linear combination of these rows (or columns).
- We can apply this fact to the matrix of state-contingent payoffs: if there are $S$ states, and $S$ linearly independent assets, then any asset can be replicated by some combination of the $S$ assets.
- If this holds, we say that the markets are *complete*.
- In an economy with complete markets, any possible risk can be insured against.
- In the real world, markets are *incomplete*, but as new types of securities and derivatives are invented, the markets are moving closer to completeness.

# Arrow securities

- The simplest asset is one that pays 1 unit in exactly one state of the world $s$, and zero in all other states.

$$e^s = (0, 0, ..., 1, ...0)^T$$

- This is called the *Arrow security* for state $s$.
- Any financial asset can be represented as a linear combination of Arrow securities.
- If we assume the *law of one price*, that is, any asset with the same state-contingent payoffs should have the same price...
- Then we should be able to find the price of any asset as a combination of prices of Arrow securities.

# Two-Period Economy

- By convention, we will say that $t = 0$ is state $s = 0$, and the states at $t = 1$ are $s = 1, 2, ..., S$.
- Let $y^s$ denote the amount of consumption in state $s$.
- Assume agents have a utility function

$$v(y^0) + \delta E[v(y)] = v(y^0) + \delta \sum_{s=1}^{S} \pi_s v(y^s)$$

- $\pi_s$ is the probability that state $s$ occurs.
- $v(\cdot)$ is a vNM utility function.
- $\delta \in (0, 1)$ is the discount factor.
- This type of utility function is *time-separable*, i.e. additive in the utility for $t = 0$ and $t = 1$.

# Efficient Risk-Sharing

- Suppose there are two agents, $S = \{1, 2\}$, and agents are risk-averse: $v^i(\cdot)$ is strictly concave.
- Agents are endowed with some amount of securities that pay off at $t = 1$.
- Assume there is no *aggregate* risk: the sum of endowments for each state $s$ is constant.
- There may be *idiosyncratic* risk: the endowment for an individual agent may differ across states.
- The *mutuality principle* states that an efficient allocation in this situation will diversify away idiosyncratic risk.
- Agents will consume the same amount in both states; they will only bear aggregate risk.

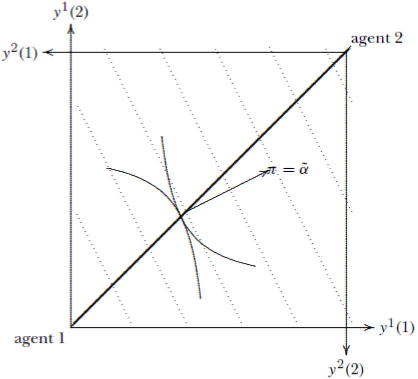



*Figure 5.1.* An Edgeworth box with no aggregate risk and full insurance. The dotted lines are iso-expected wealth lines, the fat line is the contract curve.

- Assume each agent's utility is:

$$u_i(y^i(0), y^i(1), y^i(2)) = v(y^i(0)) + \delta \sum_{s=1}^{S} \pi_s v(y^i(s))$$

- $y^i(s)$ is the amount consumed in state $s$ by agent $i$.
- Assume the same aggregate income in both states: $y^1(1) + y^2(1) = y^1(2) + y^2(2) = W$
- At equilibrium, both agents' MRS are equal to each other and the price ratio.

$$\frac{\frac{\partial u_1}{\partial y^1(1)}}{\frac{\partial u_1}{\partial y^1(2)}} = \frac{\frac{\partial u_2}{\partial y^2(1)}}{\frac{\partial u_2}{\partial y^2(2)}}$$

$$\frac{\pi_1 v_1'(y^1(1))}{\pi_2 v_1'(y^1(2))} = \frac{\pi_1 v_2'(W - y^1(1))}{\pi_2 v_2'(W - y^1(2))}$$

$$\frac{v_1'(y^1(1))}{v_2'(W - y^1(1))} = \frac{v_1'(y^1(2))}{v_2'(W - y^1(2))}$$

$$\frac{\frac{\partial u_1}{\partial y^1(1)}}{\frac{\partial u_1}{\partial y^1(2)}} = \frac{\frac{\partial u_2}{\partial y^2(1)}}{\frac{\partial u_2}{\partial y^2(2)}}$$

$$\frac{\pi_1 v_1'(y^1(1))}{\pi_2 v_1'(y^1(2))} = \frac{\pi_1 v_2'(W - y^1(1))}{\pi_2 v_2'(W - y^1(2))}$$

$$\frac{v_1'(y^1(1))}{v_2'(W - y^1(1))} = \frac{v_1'(y^1(2))}{v_2'(W - y^1(2))}$$

- By assumption, $v_1, v_2$ are strictly concave, therefore $v_1', v_2'$ are strictly decreasing.
- The function $f(x) = \frac{v_1'(x)}{v_2'(W-x)}$ is strictly decreasing, so if two values $x, x'$ give $f(x) = f(x')$, then $x = x'$.
- Therefore, $y^1(1) = y^1(2)$ and both agents consume the same amount in each state.

# Example 5.3(a)

- Suppose there are two states of the world, with probabilities $\pi, 1-\pi$ respectively.
- There are two agents with utility function of income $ln(x)$.
- Agents do not care about current consumption (in time $t = 0$), only consumption at $t = 1$.
- Therefore, we can write their utility function as

$$U(y(1), y(2)) = \pi \ln(y(1)) + (1-\pi) \ln(y(2))$$

- where $y(1), y(2)$ are consumption in state 1 and state 2, respectively.
- Suppose each agent's initial endowment is:

$$w^1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, w^2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

- We want to find the equilibrium prices and allocations.
- Note that there is no aggregate uncertainty: combined endowment is the same in state 1 and state 2.
- Therefore, we know that in equilibrium, both agents will achieve perfect insurance (that is, consume the same amount in each state).
- This is the 45-degree line on the Edgeworth box.
- Suppose $\pi = 1/2$, and let $p_1, p_2$ be the prices of the Arrow securities for state 1 and 2, respectively.
- The optimality condition $MRS_1 = MRS_2 = \frac{p_1}{p_2}$ becomes:

$$\frac{\pi y^1(2)}{(1-\pi)y^1(1)} = \frac{\pi y^2(2)}{(1-\pi)y^2(1)} = \frac{p_1}{p_2}$$

$$\frac{y^1(2)}{y^1(1)} = \frac{y^2(2)}{y^2(1)} = \frac{p_1}{p_2}$$

- Since $y^1(2) = y^1(1)$, then $p_1 = p_2$. Let's choose $p_1 = p_2 = 1$.

- What is the optimal consumption of agents 1 and 2, given prices $p_1 = p_2 = 1$?
- At these prices, agent 1's wealth is $1 \cdot 1 + 1 \cdot 3 = 4$; agent 2's wealth is $1 \cdot 3 + 1 \cdot 1 = 4$.
- The optimal consumption for both agents is $y^1 = 2, y^2 = 2$.
- This also satisfies the market clearing condition, since the total amount of $y^1$ consumed by both agents is equal to the total initial endowment (and the same holds for $y^2$).
- Therefore, the Walrasian equilibrium is the combination of price and allocation:
  - prices: any $p_1, p_2$ that satisfies $p_1 = p_2$
  - allocation: agent 1's consumption is $(2, 2)$, agent 2's consumption is $(2, 2)$
- The two conditions for a Walrasian equilibrium are satisfied:
  - Both agents are choosing the utility maximizing choice, given prices $p_1, p_2$
  - Market clearing is satisfied: for each of the goods $y(1), y(2)$, total consumption is equal to total endowment

- Note that both agents are perfectly insured (that is, consume the same amount in each state).
- The agents are insuring each other.
- This is only possible because the "good" state for agent 1 is the "bad" state for agent 2, and vice versa.
- This is an example of *mutual insurance*: a group of consumers pool their resources to insure each other, instead of going to an outside party like an insurance company.

# Mutuality Principle

- **Lengwiler Box 5.1 (Mutuality Principle)**: An efficient allocation of risk requires that only aggregate risk be borne by agents. All idiosyncratic risk is diversified away by mutual insurance among agents.
- The marginal aggregate risk borne by an agent equals the ratio of his absolute risk tolerance to the average risk tolerance of the population.
- The mutuality principle can fail if:
  - Beliefs are heterogeneous (different agents have different subjective probabilities of states)
  - if market frictions (e.g. trading costs, short sale constraints) impede Pareto efficiency
  - if markets are incomplete

# Mutuality Principle

- This principle has many applications in different fields of economics.
    - In international macro, many papers try to test efficient risk sharing among different countries, and explain if/why it does not occur
    - In labor, test efficient risk sharing among workers, retirees, health insurance consumers, etc
- Many of the models we have seen in this course want to explain banks as a way to implement some sort of risk-sharing.
- However, risk-sharing is not the only motivation for financial transactions.

- What happens if agent 1 is risk-averse, but agent 2 is risk-neutral?
- For example, suppose agent 1 and agent 2 have utility functions (where $v$ is a concave function):

$$U_1(y^1(1), y^1(2)) = \pi v(y^1(1)) + (1-\pi)v(y^1(2))$$

$$U_2(y^2(1), y^2(2)) = \pi y^2(1) + (1-\pi)y^2(2)$$

.

- Agent 2's indifference curves will be linear.
- The condition $MRS_1 = MRS_2$ will become:

$$\frac{\pi v'(y^1(1))}{(1-\pi)v'(y^1(2))} = \frac{\pi}{1-\pi}$$

$$\frac{\pi v'(y^1(1))}{(1-\pi)v'(y^1(2))} = \frac{\pi}{1-\pi}$$

- By the same argument as before, agent 1 will fully insure: $y^1(1) = y^1(2)$.
- Note that we are no longer assuming that there is no aggregate uncertainty: the total endowment of each good may be different in different states.
- This means that in equilibrium, the risk-neutral agent is assuming all the risk, and the risk-averse agent has shifted all of his risk away.

# Next Week

- Homework 1 is due next week.
- Next week, we will start a brief introduction to game theory. Please read Chapter 2.1-2.8 in Osborne.